

Results of the $K_L \rightarrow \pi^+\pi^-e^+e^-$ Analysis

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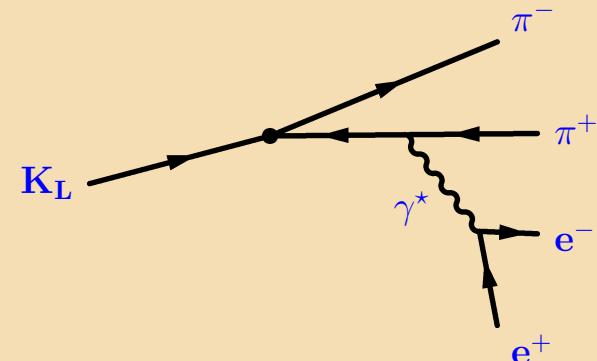
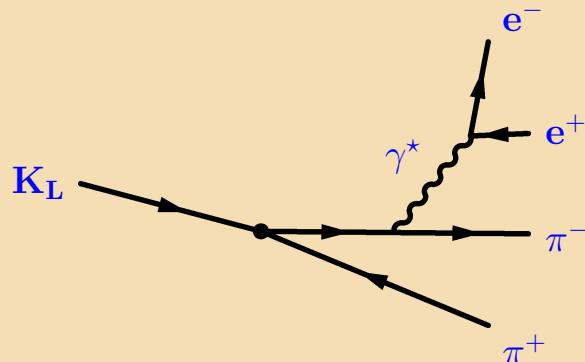
KTev Collaboration Meeting
*Fermilab**

*In this document (if you have access to the internet) you can click on any text in *purple* color for additional information

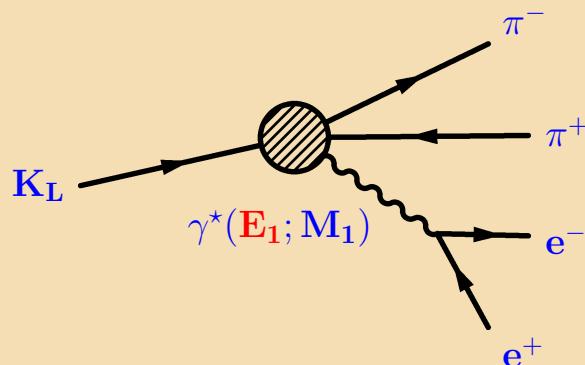
1. What Am I Going to Talk About?

- ✓ Include simulation of **radiative corrections in MC**.
- ✓ **Final Systematic** Uncertainties.
- ✓ Final results from the fit: g_{CR} , \tilde{g}_{M_1} , $\frac{a_1}{a_2}$ and $\frac{|g_{E_1}|}{|g_{M_1}|}$
- ✓ **Final result** for the asymmetry \mathcal{A} .

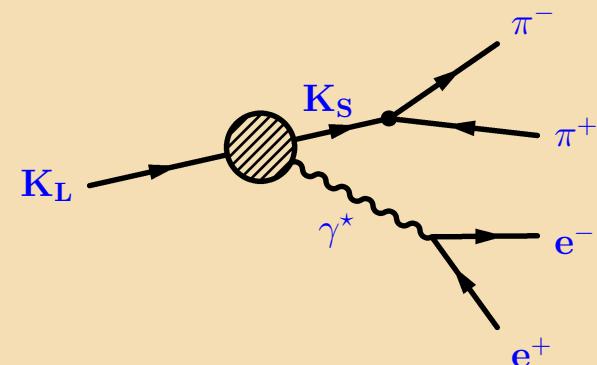
2. Contributions to the Decay $K_L \rightarrow \pi^+ \pi^- e^+ e^-$



Inner Bremsstrahlung (IB) — *Indirect QP*



Direct Emission (DE)
 E_1 — *Indirect QP*
 M_1 — *CP Conserving*



K^0 Charge Radius (CR)
CP Conserving

3. The Likelihood Function

- ✓ Matrix element $\mu = \mu(x; \alpha, \beta)$ represents **our model** for $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ decay, where

$$x = (\phi, \cos\theta_{e^+}, \cos\theta_{\pi^-}, M_{\pi\pi}, M_{ee}), \quad \alpha = \left(\frac{a_1}{a_2}; \tilde{g}_{M_1}; g_{CR}; g_{E_1} \right)$$

and β defines all other parameters.

- ✓ We use the method of **maximum likelihood** with re-weighting to extract the parameters α . Logarithm of the likelihood function \mathcal{L} can be written as follows:

$$\log \mathcal{L}(\alpha) = \sum_{i=1}^{N_d} \log \mu(x_i; \alpha) - N_d \cdot \log \sum_{j=1}^{N_{mc}} \frac{\mu(x_j; \alpha)}{\mu(x_j; \alpha_0)} + f(x, \alpha_0)$$

where f does not depend on α .

- ✓ We generate **one large MC sample** for a certain initial choice of $\alpha = \alpha_0$ and then re-weight it to obtain $\mathcal{L}(\alpha)$ for all other values of α .

4. Expressions for the Form Factors

- ✓ Inner Bremsstrahlung: $g_{IB} = |\eta_{+-}| e^{i(\delta_0(M_K) + \Phi_{+-})}$
- ✓ M_1 Direct Emission: $g_{M_1} = i e^{i\delta_1(M_{\pi\pi})} \times |g_{M_1}|,$

where

$$|g_{M_1}| \equiv \tilde{g}_{M_1} \left[1 + \frac{a_1/a_2}{(M_\rho^2 - M_K^2) + 2M_K E_{ee}} \right]$$

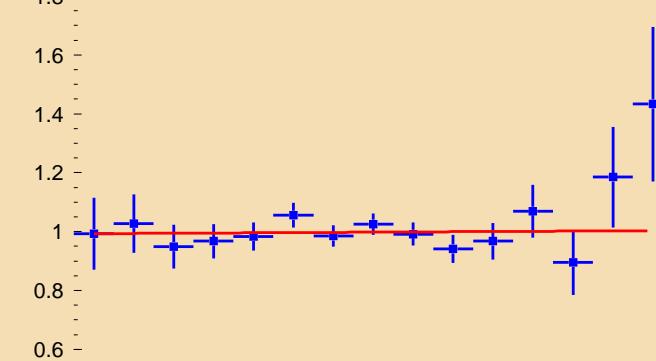
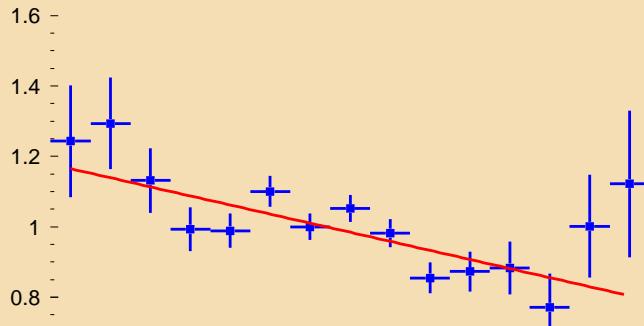
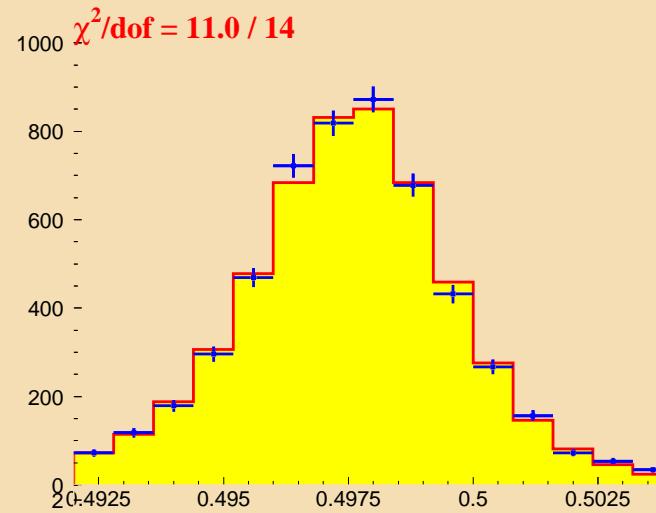
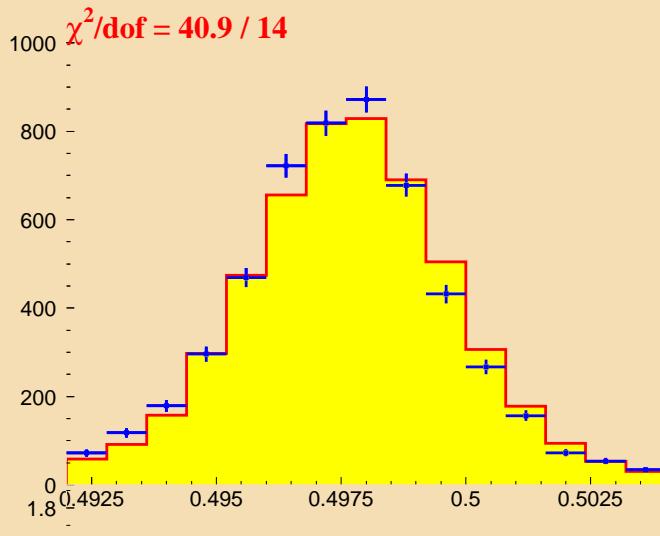
- ✓ E_1 Direct Emission: $g_{E_1} = \frac{|g_{E_1}|}{|g_{M_1}|} e^{i(\delta_1(M_{\pi\pi}) + \Phi_{+-})} \times |g_{M_1}|$
- ✓ Charge Radius: $g_{CR} = |g_{CR}| e^{i\delta_0(M_{\pi\pi})},$

where $|g_{CR}| = -\frac{1}{3} \langle R^2(K^0) \rangle M_K^2$

5. Radiative Corrections in the MC

- ✓ PHOTOS simulates radiation of real soft photons.
- ✓ Use default PHOTOS configuration.
- ✓ Patch it to ktevmc v5_06 code.
- ✓ Use 1 KeV infrared cutoff.

6. PHOTOS Improves DATA/MC overlay for $M_{\pi\pi ee}$



χ^2/ndf	19.87 / 13
P1	17.03 ± 0.2330

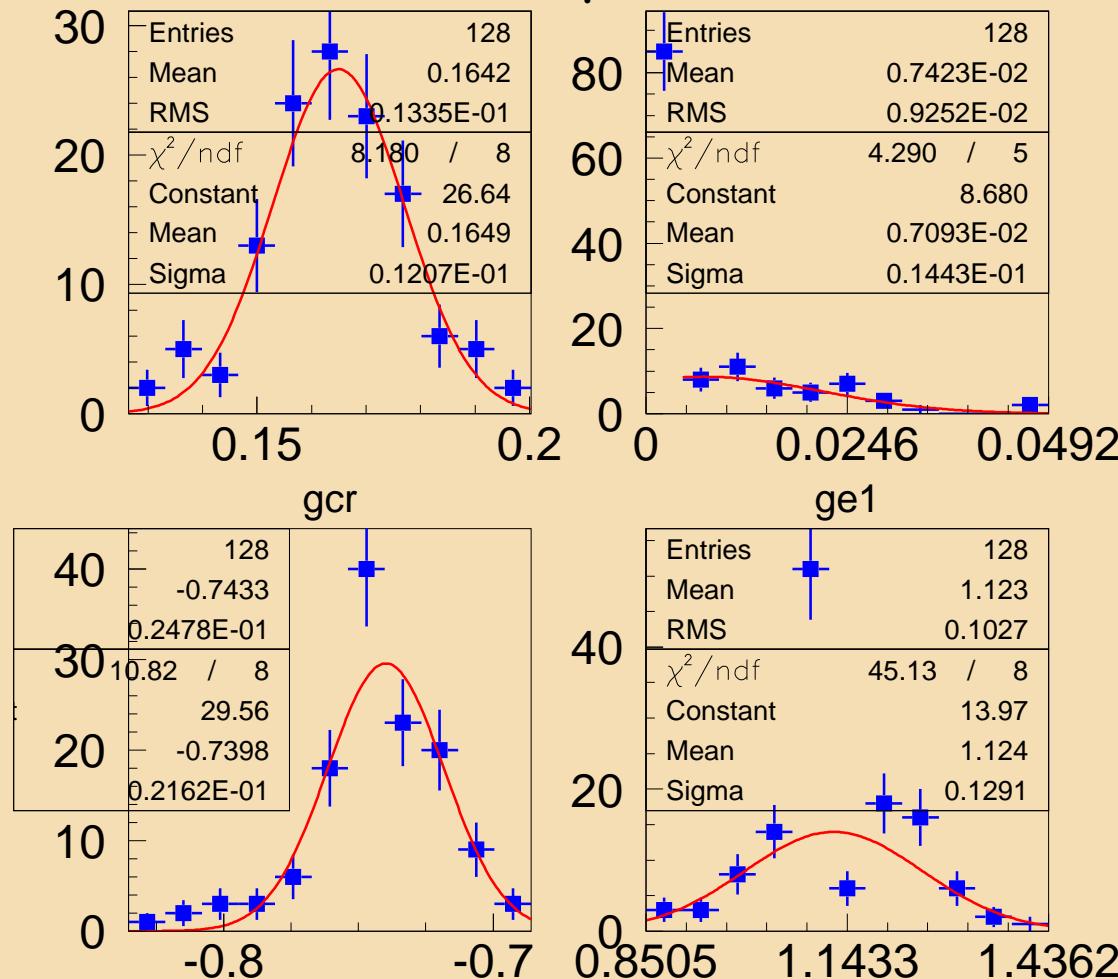
Without PHOTOS

χ^2/ndf	10.77 / 13
P1	0.5447 ± 0.2287

With PHOTOS in MC

7. There Is No Bias In The Fit Results With The New MC.

$g_{CRin} = .163$, $g_{E_1in} = .0001$, $g_{M_1in} = 1.111$, $\frac{a_1}{a_2}in = -.744$
 $g_{CRout} = .164$, $g_{E_1out} = .007$, $g_{M_1out} = 1.123$, $\frac{a_1}{a_2}out = -.743$
(blue: distributions for fake data, red: their fit to a Gaussian)



8. Effect Of PHOTOS On The Fit Results Is Small

✓ Results of the 4 parameter fit **without PHOTOS** in MC

$$g_{\text{CR}} = 0.159 \pm 0.001(\text{MCstat})$$

$$\frac{|g_{E_1}|}{|g_{M_1}|} = 0.00005 \pm 0.001(\text{MCstat})$$

$$\frac{a_1}{a_2} = -0.730 \pm 0.002(\text{MCstat})$$

$$\tilde{g}_{M_1} = 1.15 \pm 0.01(\text{MCstat})$$

✓ Results of the 4 parameter fit **with PHOTOS** in MC

$$g_{\text{CR}} = 0.163 \pm 0.001(\text{MCstat})$$

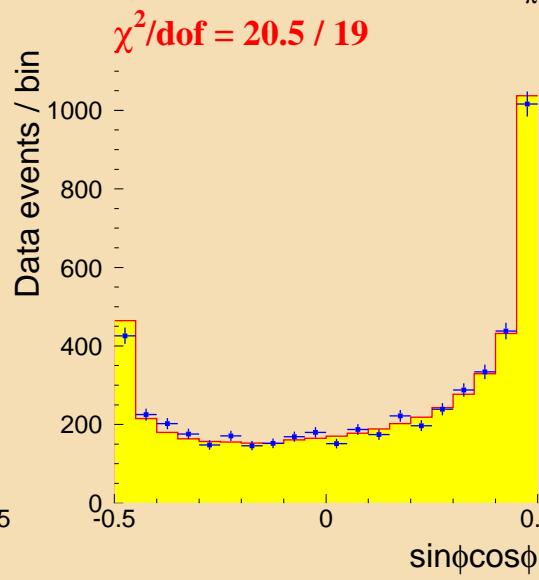
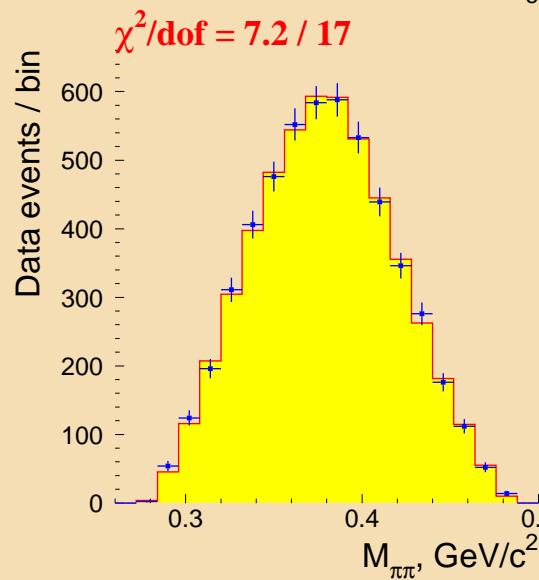
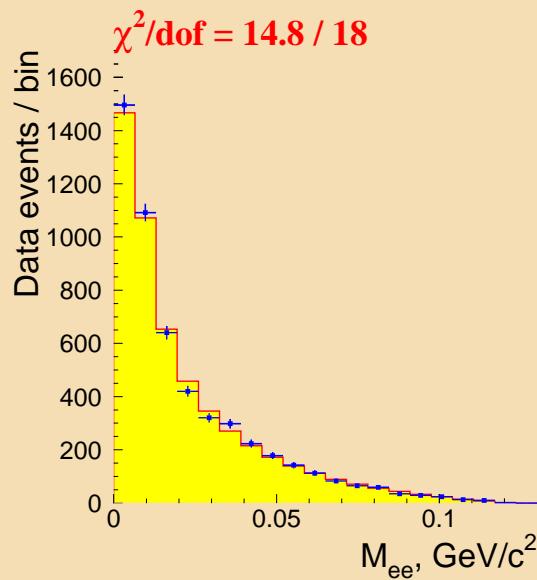
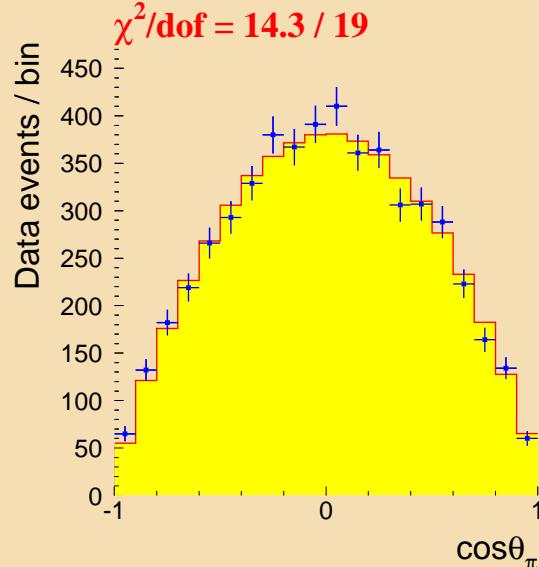
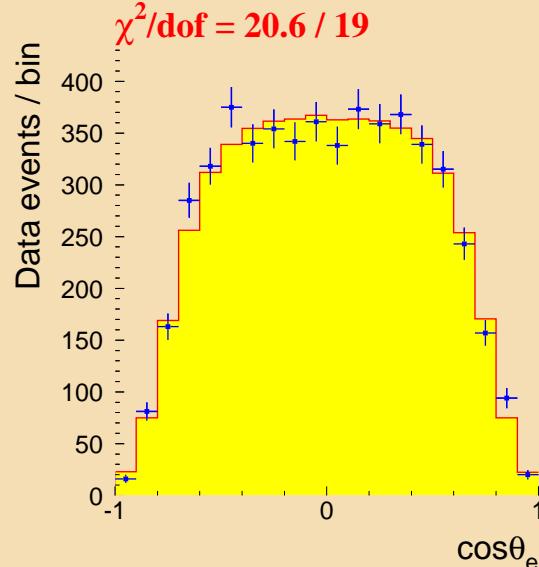
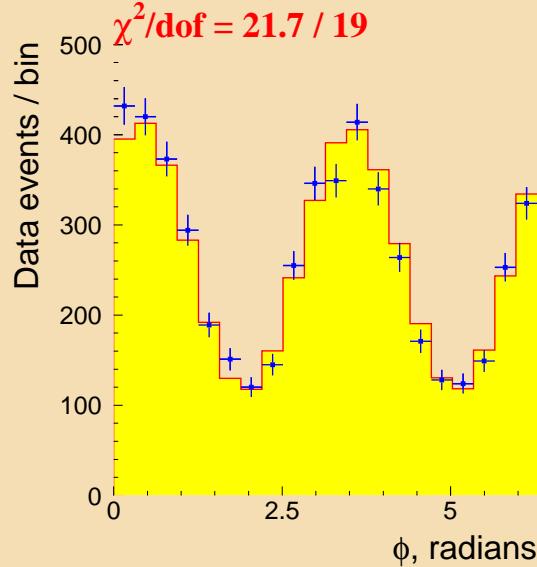
$$\frac{|g_{E_1}|}{|g_{M_1}|} = 0.0006 \pm 0.001(\text{MCstat})$$

$$\frac{a_1}{a_2} = -0.745 \pm 0.002(\text{MCstat})$$

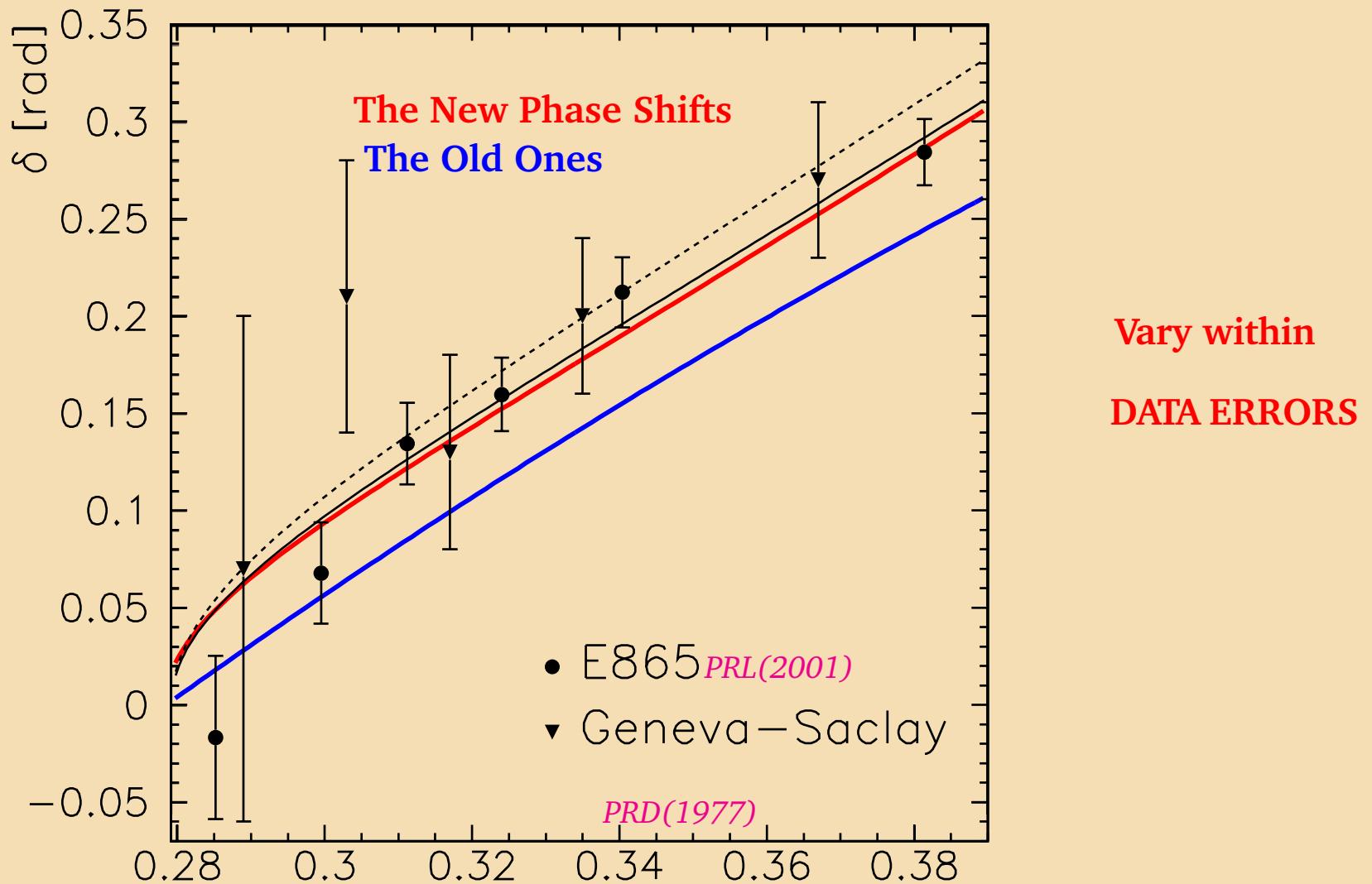
$$\tilde{g}_{M_1} = 1.10 \pm 0.01(\text{MCstat})$$

✓ Results “with PHOTOS” will be quoted. No systematic will be assigned.

9. DATA/MC Overlays: The 5 Variables + $\sin\phi \cdot \cos\phi$



9.1. Include $\Delta\delta$'s Uncertainty Estimation

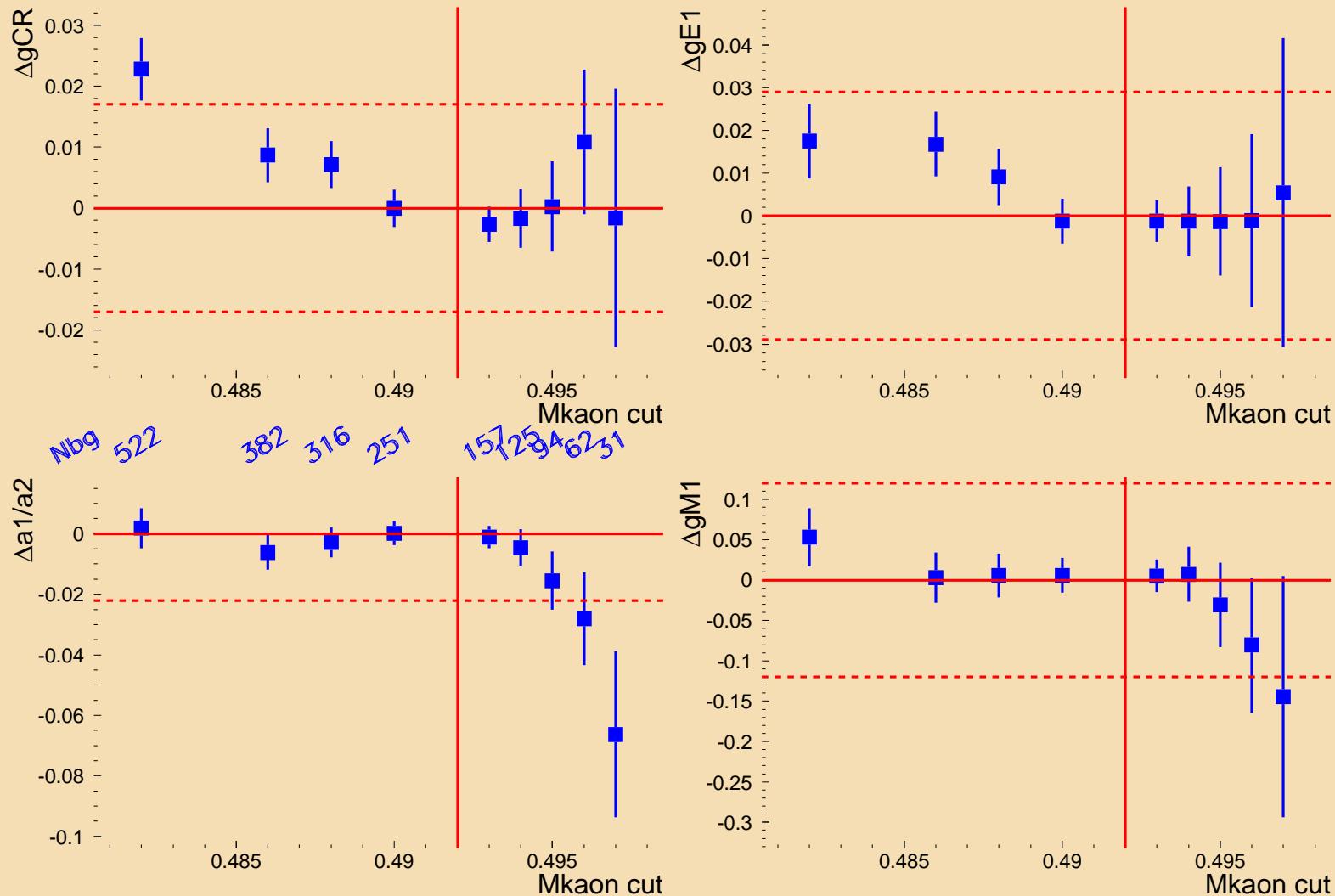


10. Study Systematic Effects with Cut Variation.

- ✓ To estimate the effect of the background we vary 3 cuts: kaon mass, P_T^2 and pp0kine, and assign the systematic uncertainty.
- ✓ Then vary other cuts and assign systematic uncertainty by adding individual contributions in quadrature.

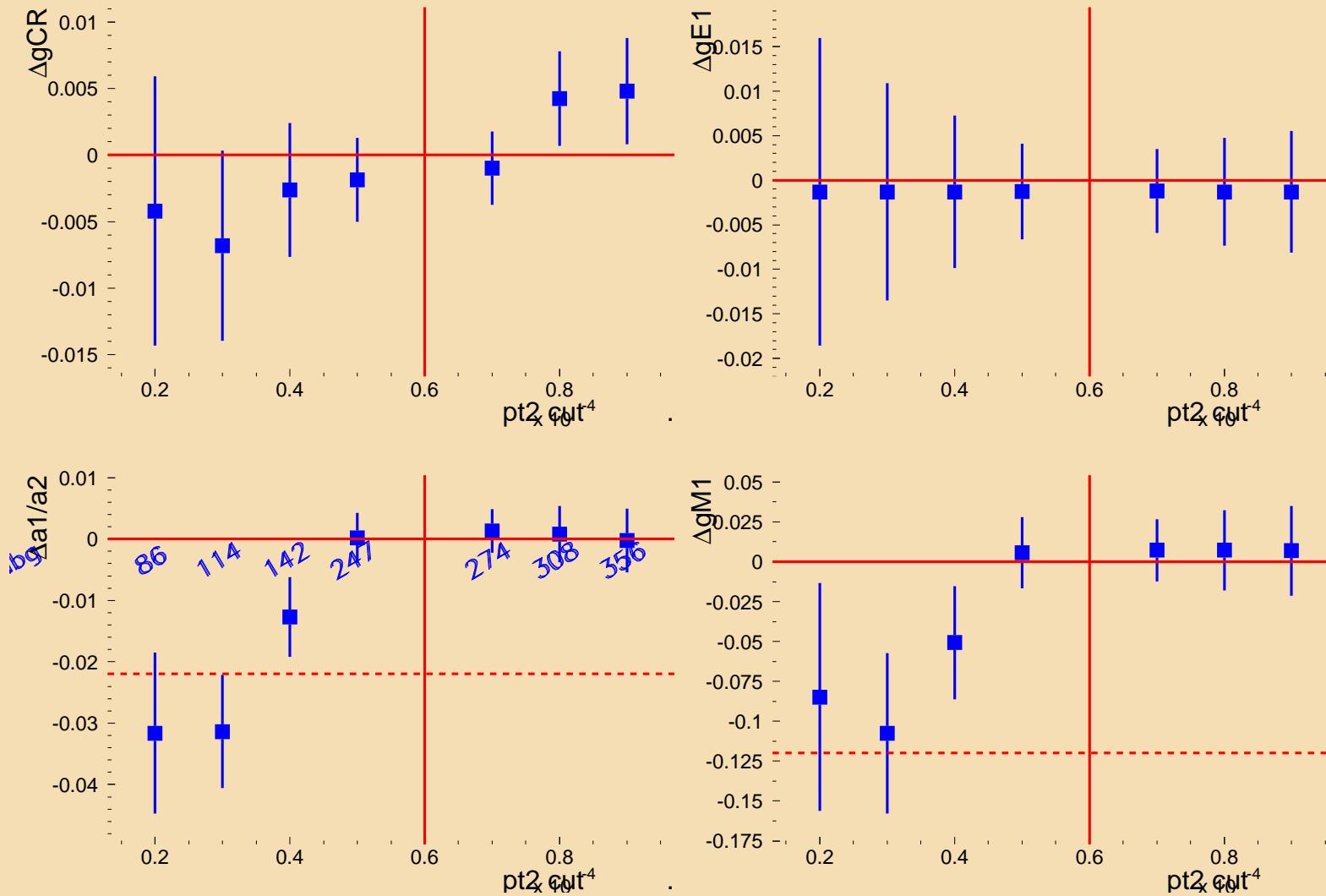
11. Vary Kaon Mass Cut: estimated N_{bg} is written in blue.

Estimated shifts: $\Delta g_{CR}=0.003$, $\Delta g_{E1}=0.008$, $\Delta a_{12}=0.01$, $\Delta g_{M1}=0$



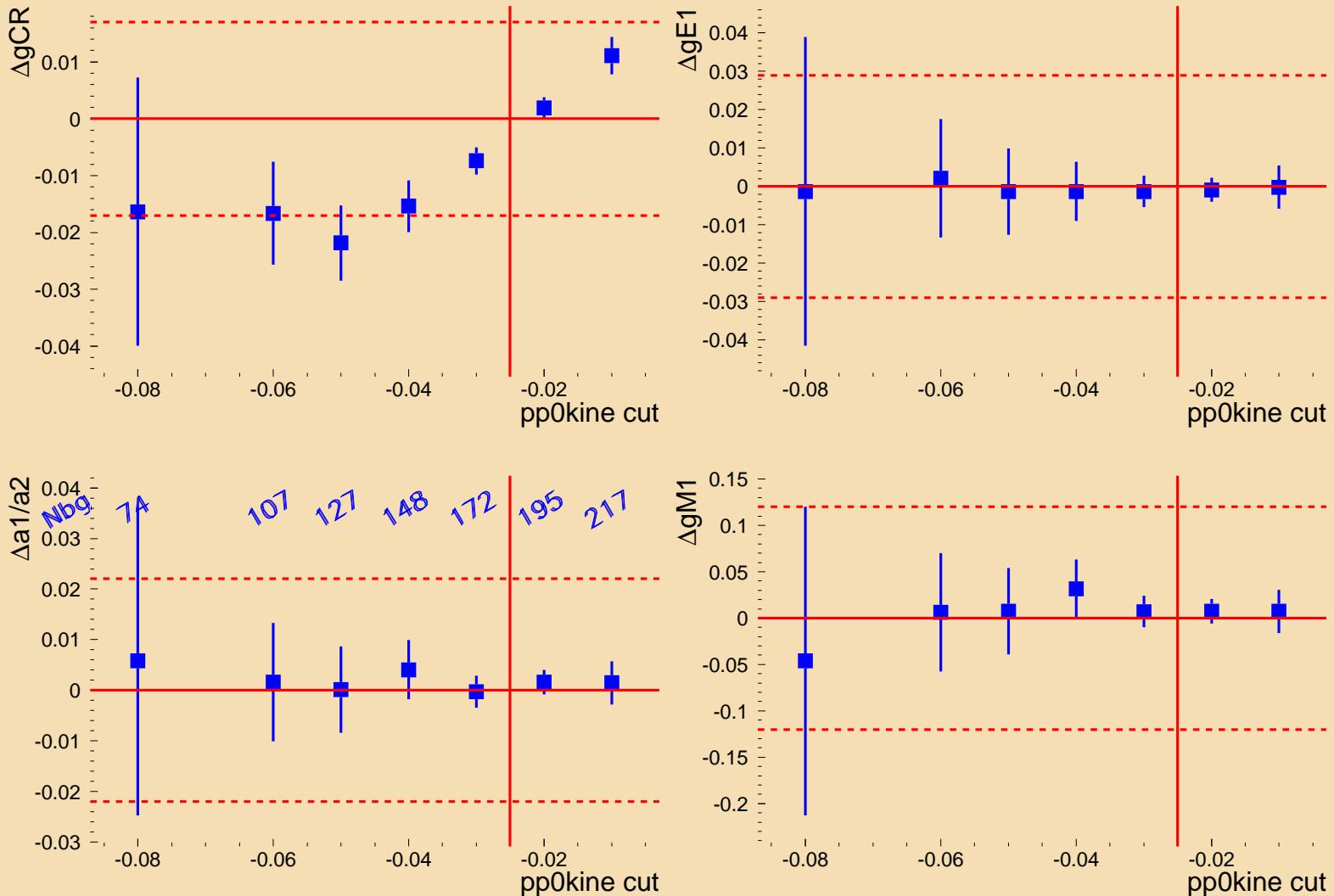
12. Vary P_T^2 Cut: estimated N_{bg} is written in blue.

Estimated shifts: $\Delta g_{CR}=0$, $\Delta g_{E1}=0$, $\Delta a_{12}=0.02$, $\Delta g_{M1}=0.05$



13. Vary p0kine Cut: estimated N_{bg} is written in blue.

Estimated shifts: $\Delta g_{CR}=0.01$, $\Delta g_{E1}=0$, $\Delta a_{12}=0$, $\Delta g_{M1}=0$

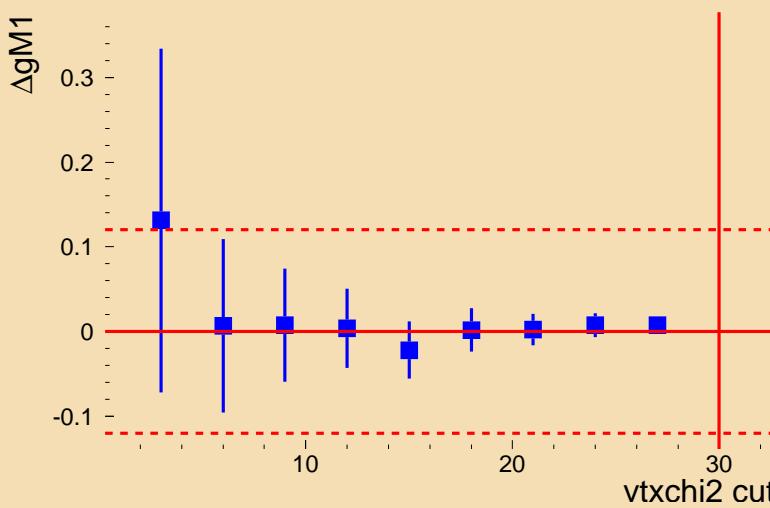
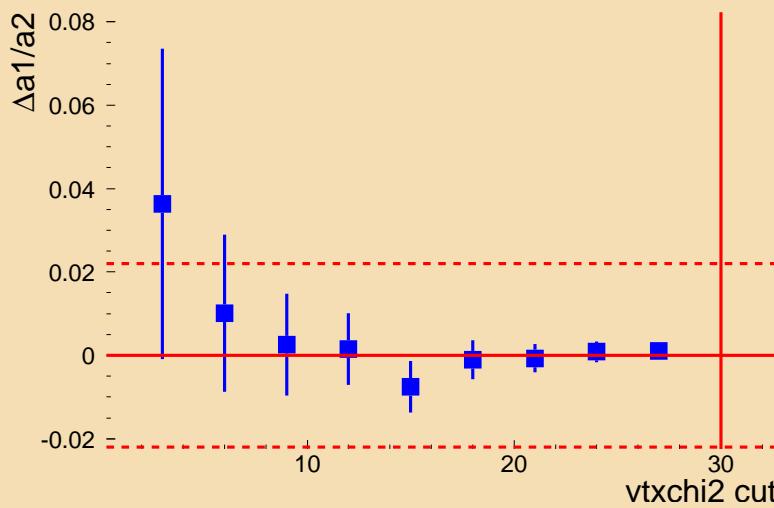
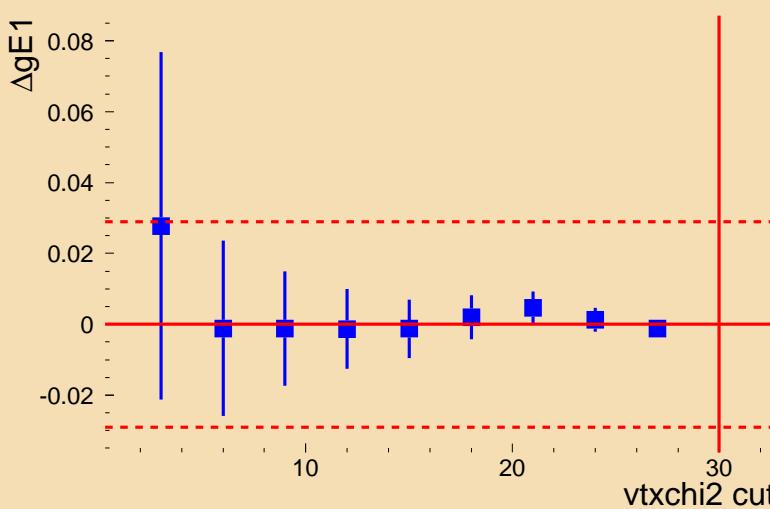
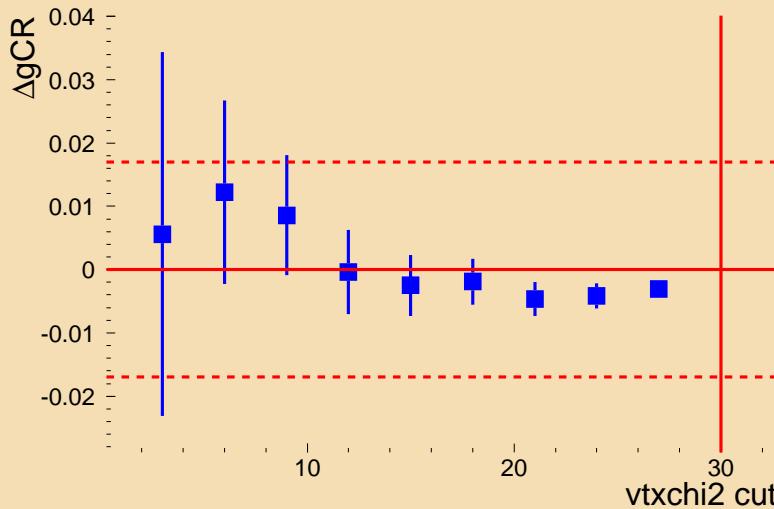


14. Summarize Background Systematics

Cut	Estimated Shift			
	Δg_{CR}	$\Delta \frac{ g_{E_1} }{ g_{M_1} }$	$\Delta \frac{a_1}{a_2}$	Δg_{M_1}
Kaon Mass	0.003	0.008	0.01	0.0
P_T^2	0.0	0.0	0.02	0.05
pp0kine	0.01	0.0	0.0	0.0
Assigned Error	0.01	0.008	0.022	0.05
Compare to Stat.	0.017	0.028	0.022	0.12

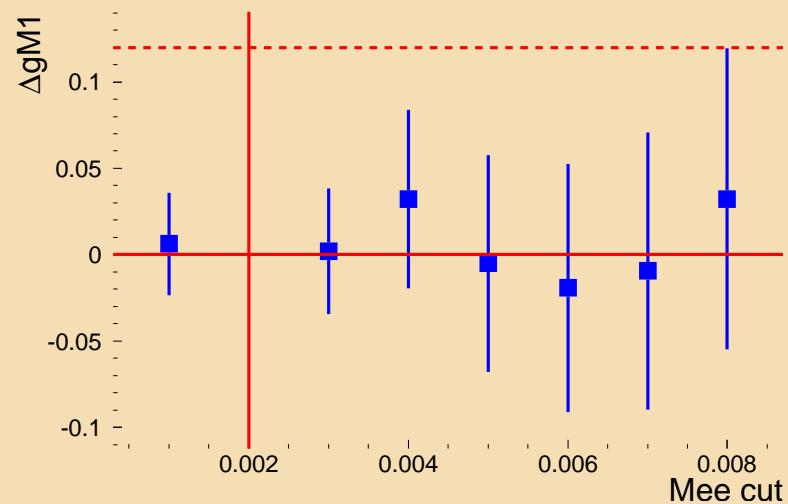
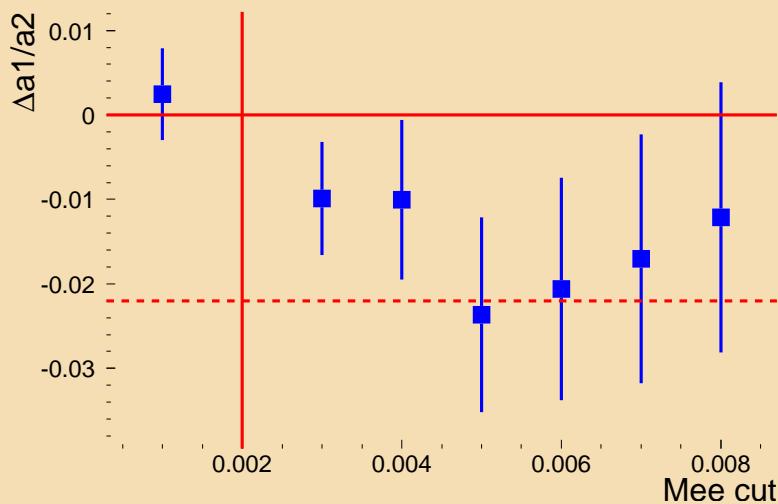
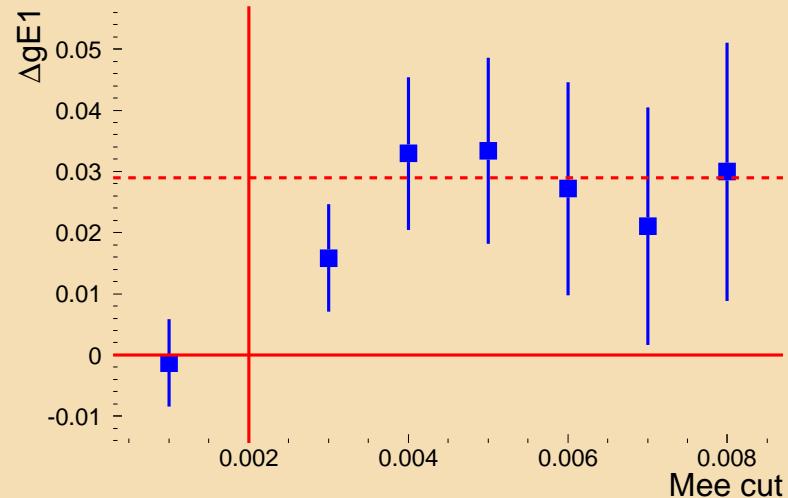
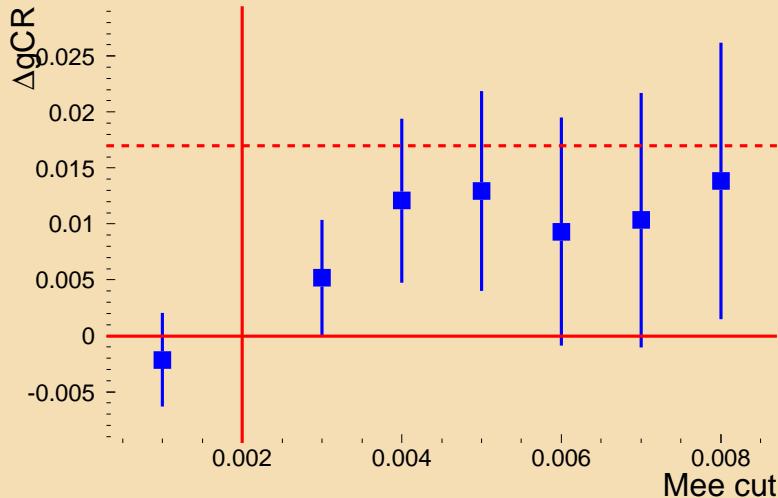
15. Vary χ^2_{vtx} Cut

Estimated shifts: $\Delta g_{CR}=0.002$, $\Delta g_{E1}=0$, $\Delta a_{12}=0$, $\Delta g_{M1}=0$



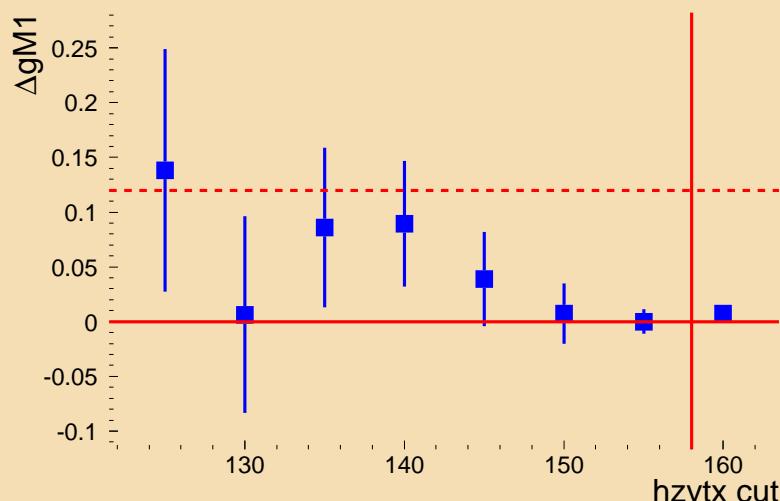
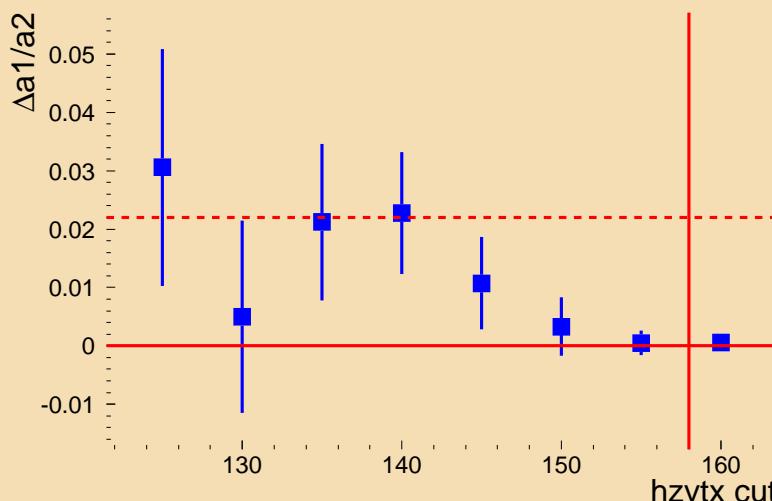
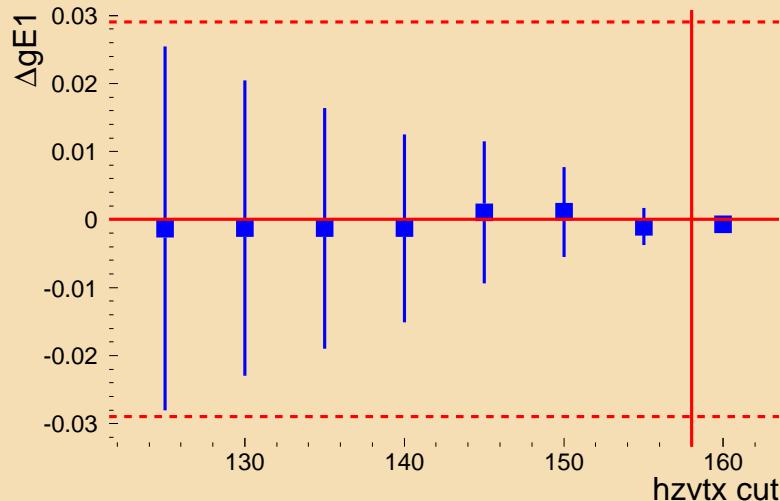
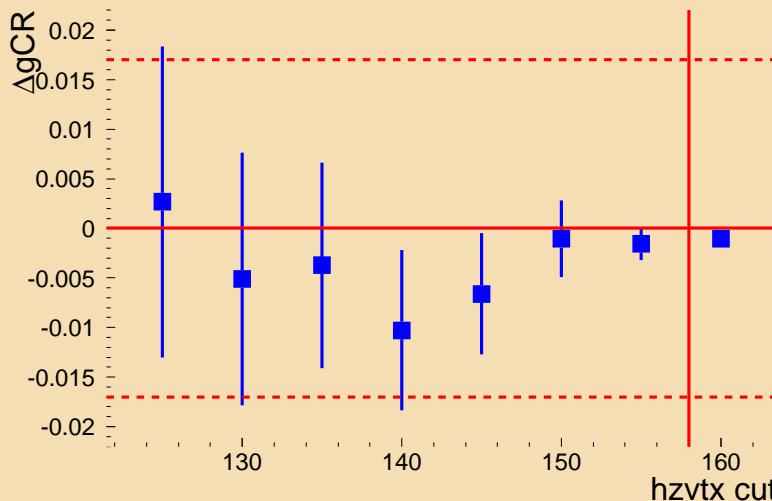
16. Vary M_{ee} Cut

Estimated shifts: $\Delta g_{CR}=0.005$, $\Delta g_{E1}=0.01$, $\Delta a_{12}=0.01$, $\Delta g_{M1}=0$



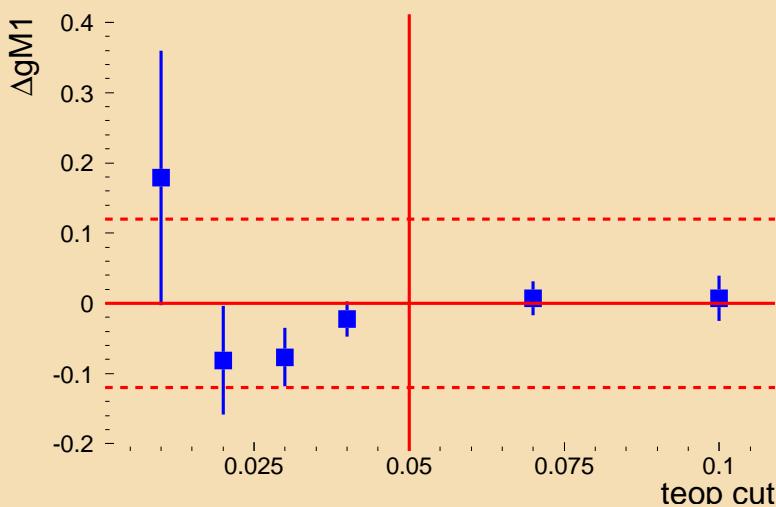
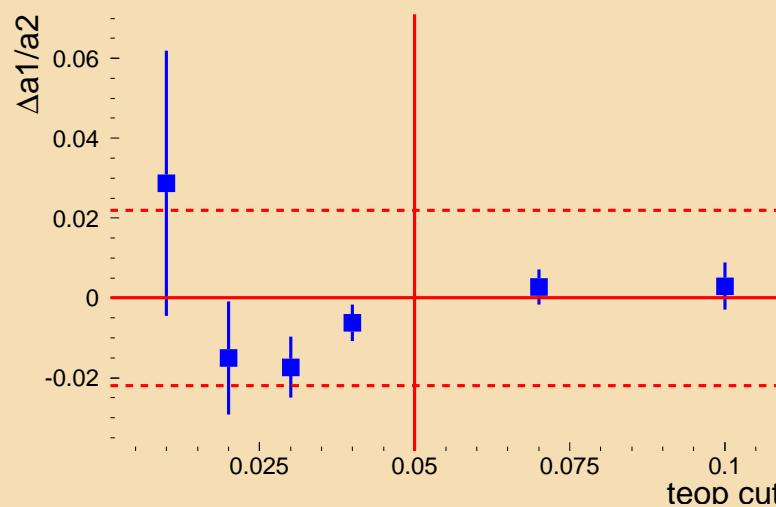
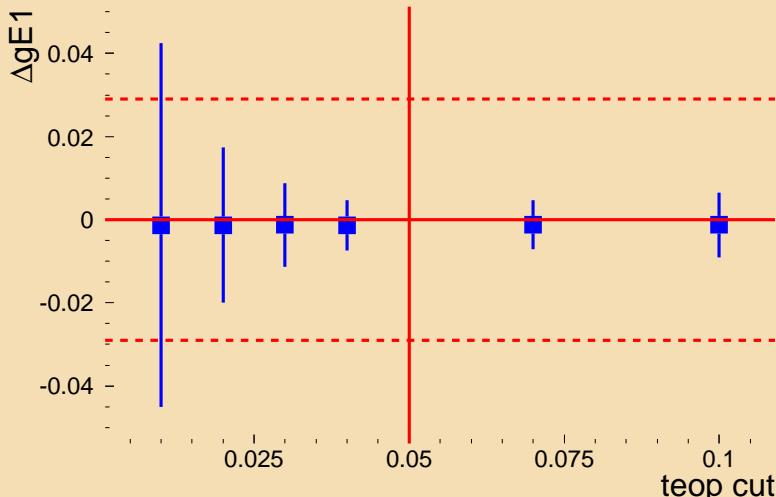
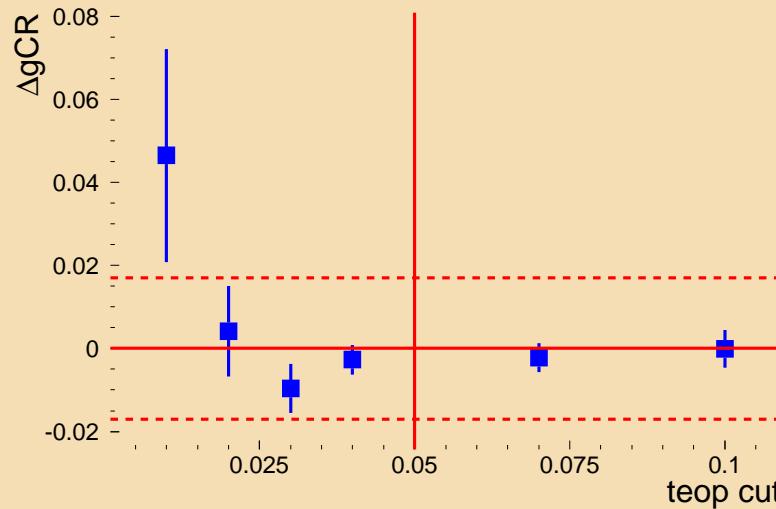
17. Vary Higher Edge of Z_{vtx} Cut

Estimated shifts: $\Delta g_{CR}=0$, $\Delta g_{E1}=0$, $\Delta a_{12}=0.01$, $\Delta g_{M1}=0.02$



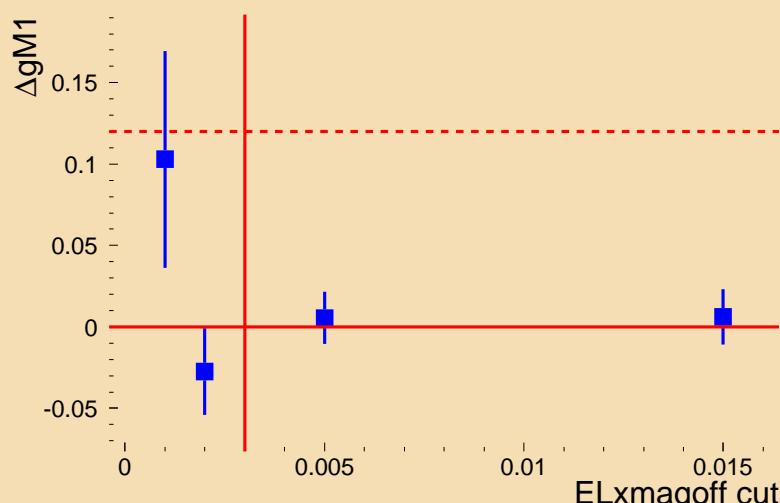
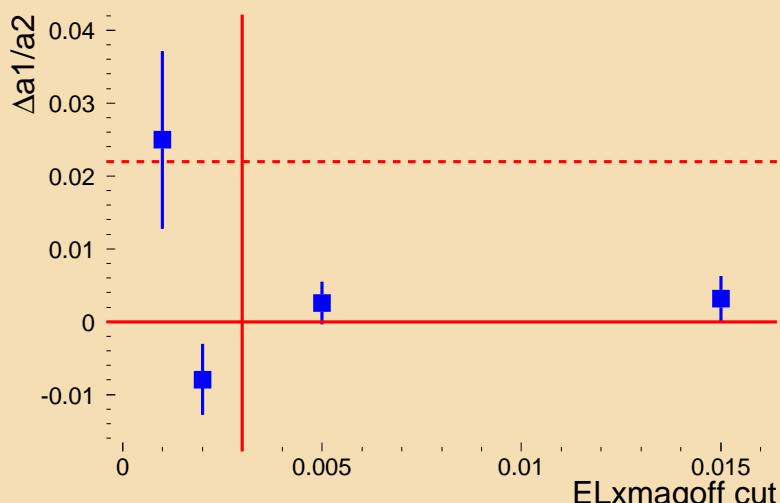
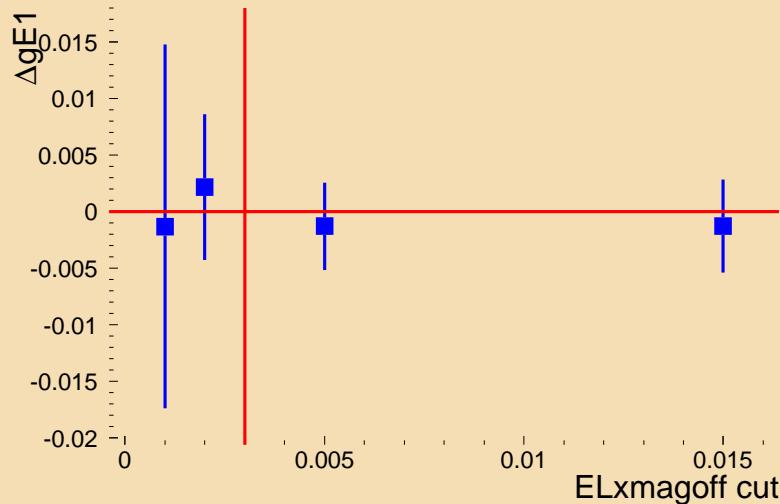
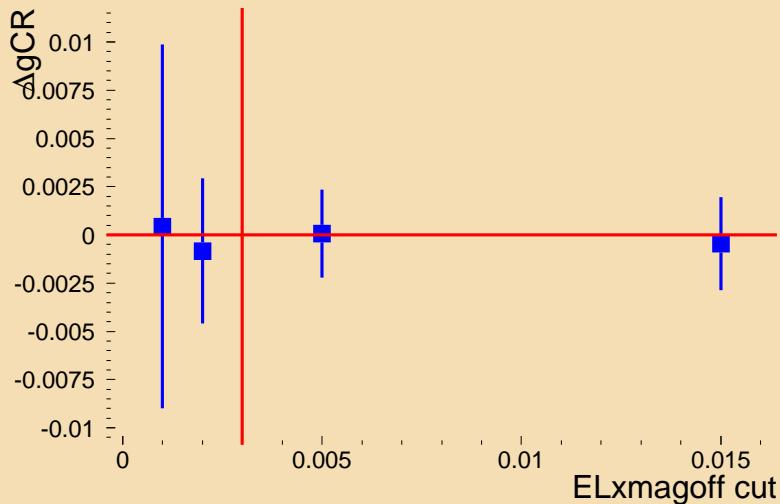
18. Vary E/P Cut

Estimated shifts: $\Delta g_{CR}=0$, $\Delta g_{E1}=0$, $\Delta a_{12}=0.005$, $\Delta g_{M1}=0.02$



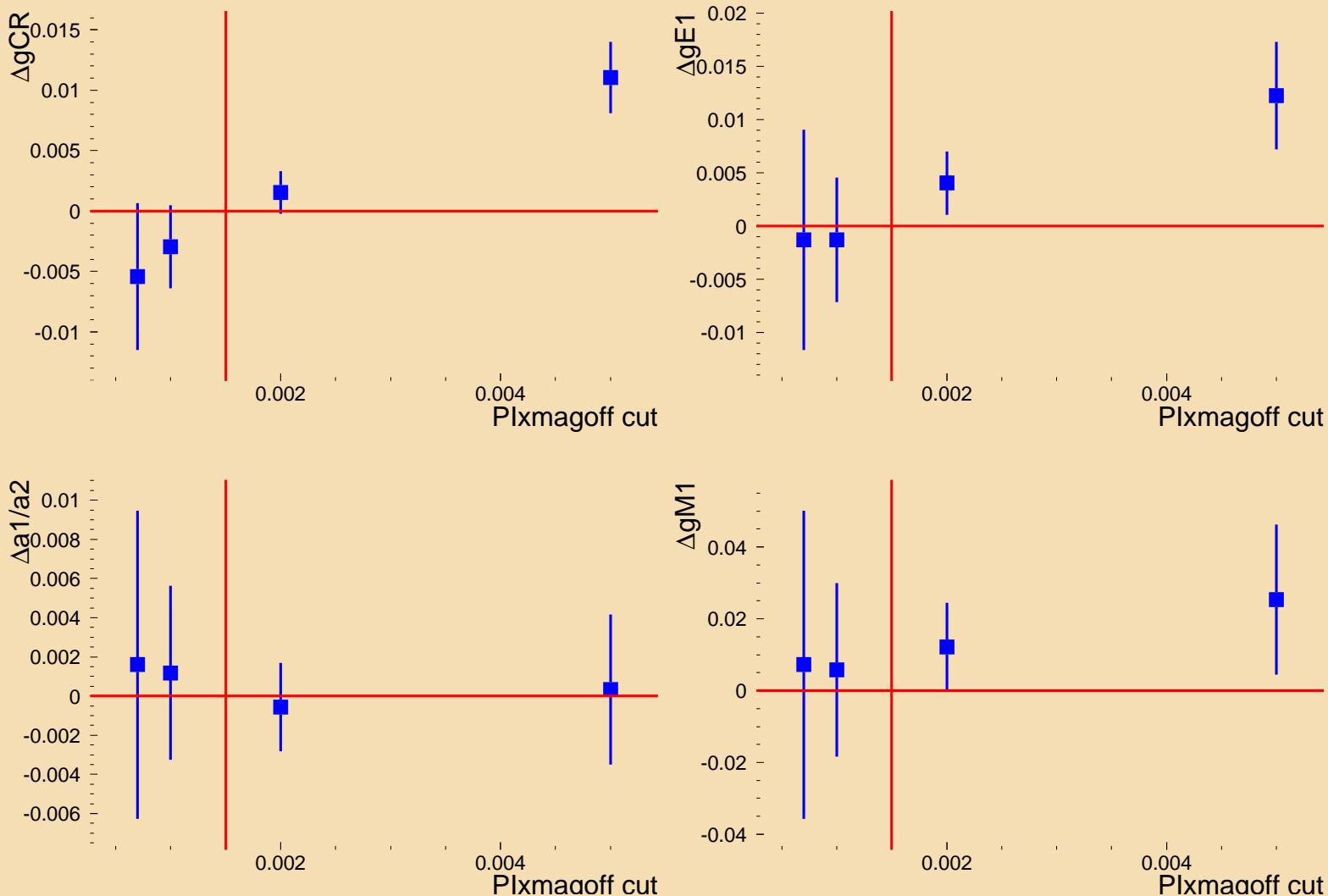
19. Vary Cut on the Magnet Offset of Electron

Estimated shifts: $\Delta g_{CR}=0$, $\Delta g_{E1}=0$, $\Delta a_{12}=0$, $\Delta g_{M1}=0$



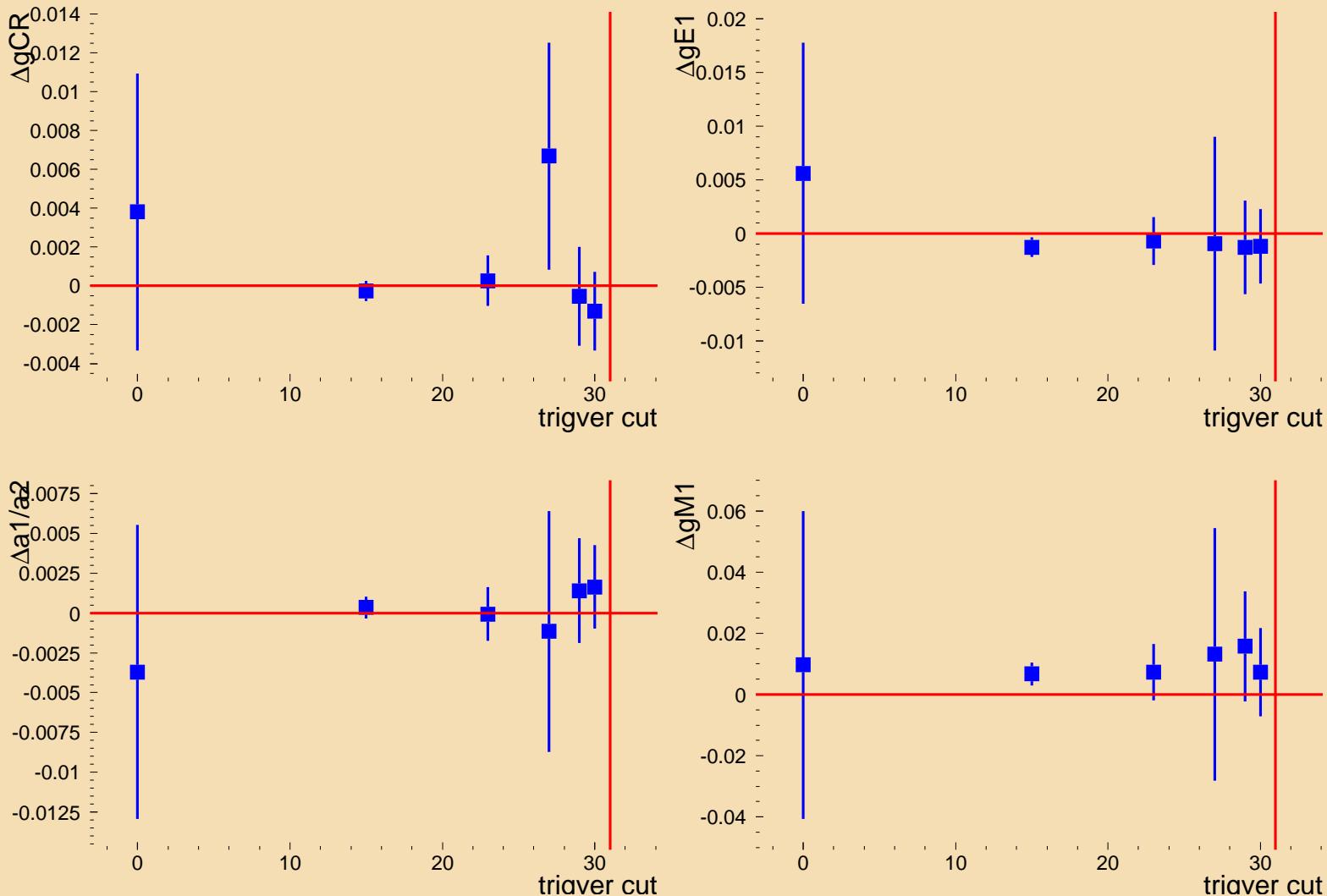
20. Vary Cut on the Magnet Offset of Pion

Estimated shifts: $\Delta g_{CR}=0$, $\Delta g_{E1}=0$, $\Delta a_{12}=0$, $\Delta g_{M1}=0$



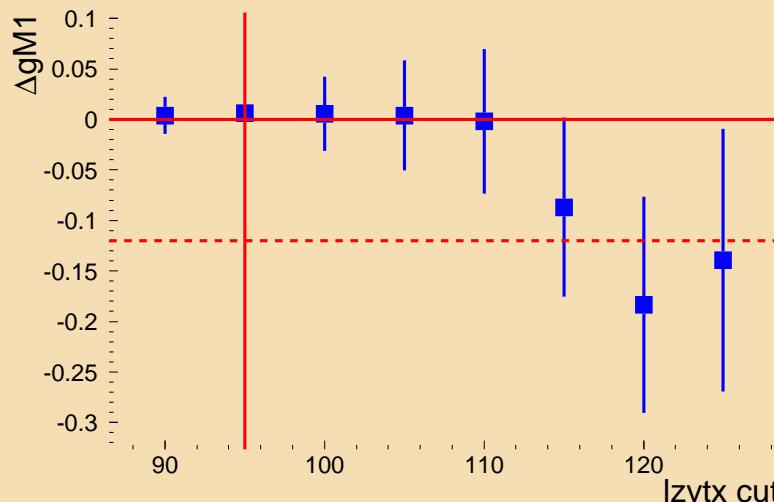
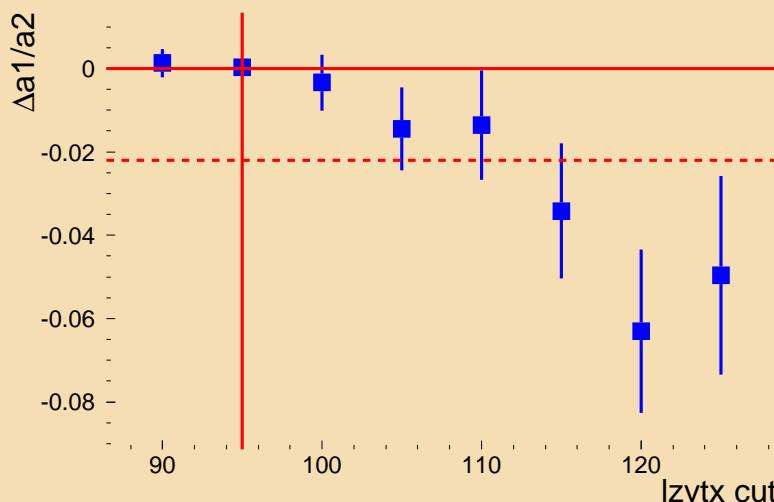
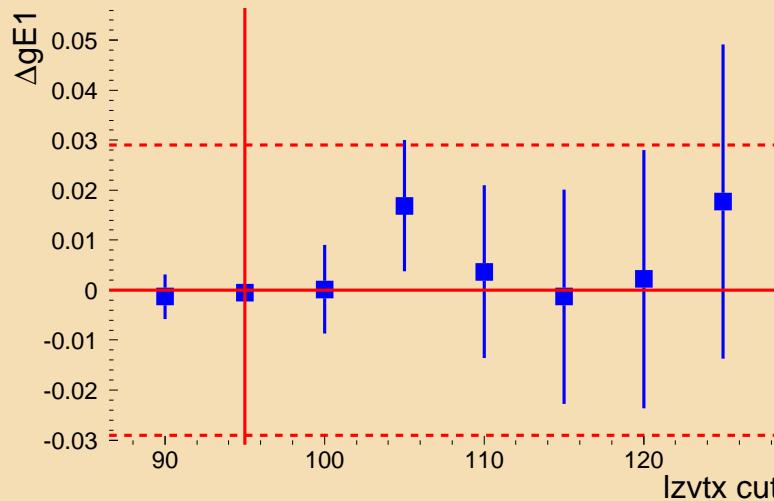
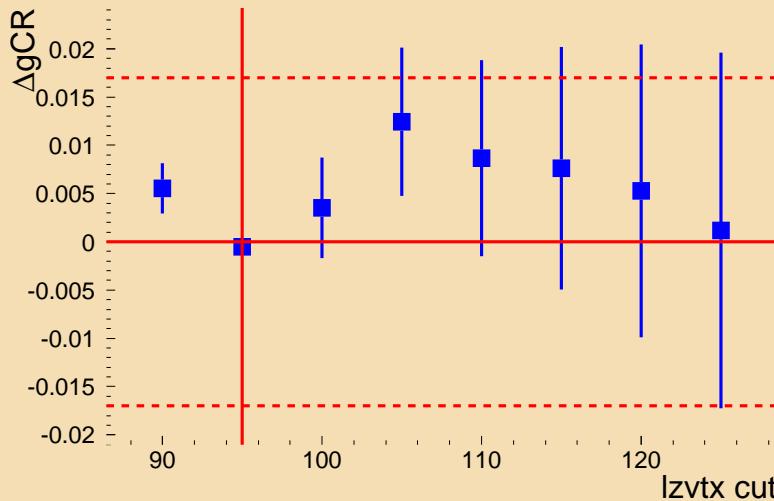
21. Vary Trigger Verification Requirements

Estimated shifts: $\Delta g_{CR}=0$, $\Delta g_{E1}=0$, $\Delta a_{12}=0$, $\Delta g_{M1}=0$



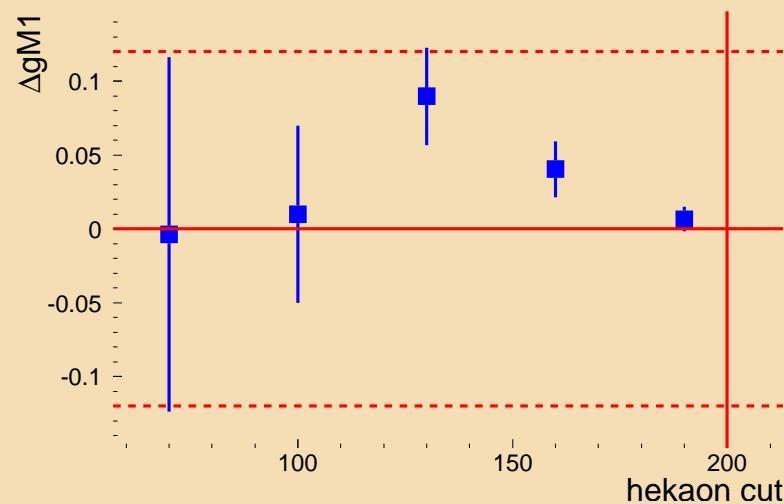
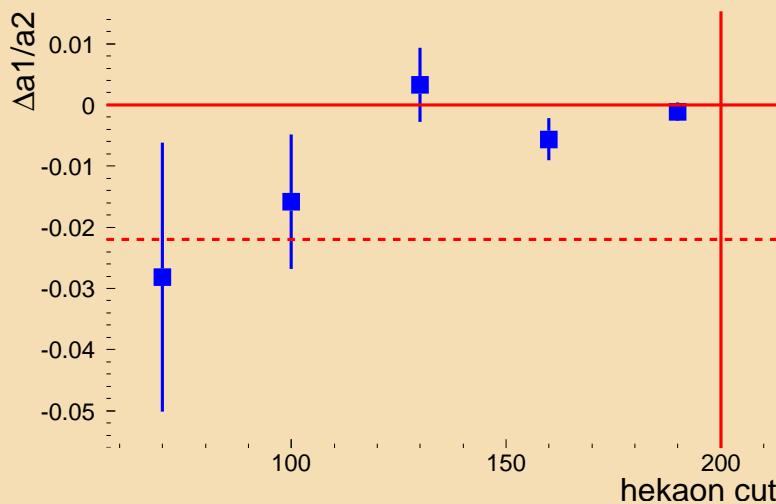
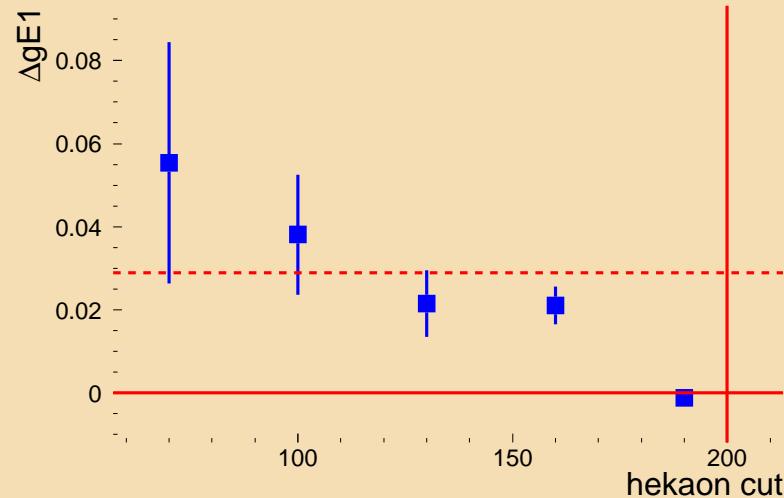
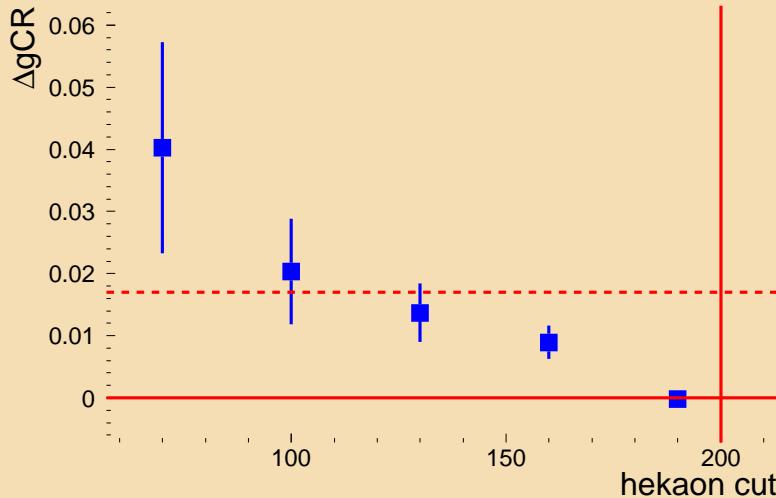
22. Vary Lower Edge of Z_{vtx} Cut

Estimated shifts: $\Delta g_{CR}=0$, $\Delta g_{E1}=0$, $\Delta a_{12}=0.02$, $\Delta g_{M1}=0$



23. Vary Kaon Energy Cut

Estimated shifts: $\Delta g_{CR}=0.02$, $\Delta g_{E1}=0.015$, $\Delta a_{12}=0$, $\Delta g_{M1}=0.03$



24. Summarize Cut Variation Systematics

Cut	Estimated Shift			
	Δg_{CR}	$\Delta \frac{ g_{E_1} }{ g_{M_1} }$	$\Delta \frac{a_1}{a_2}$	Δg_{M_1}
χ^2_{vtx}	0.002	0.0	0.0	0.0
M_{ee}	0.005	0.01	0.01	0.0
Z_{vtx}	0.0	0.0	0.02	0.02
E/P	0.0	0.0	0.005	0.02
E_{kaon}	0.02	0.015	0.0	0.03
Assigned Error	0.021	0.018	0.022	0.041
Compare to Stat.	0.017	0.028	0.022	0.12

25. Summarize All Systematics

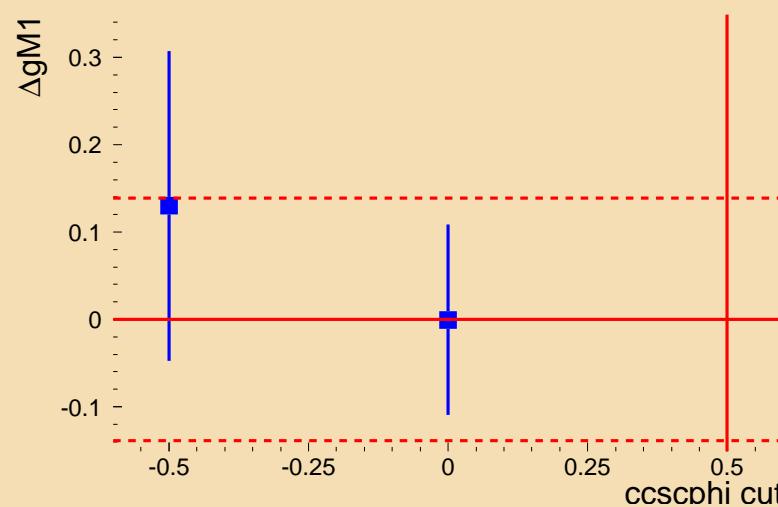
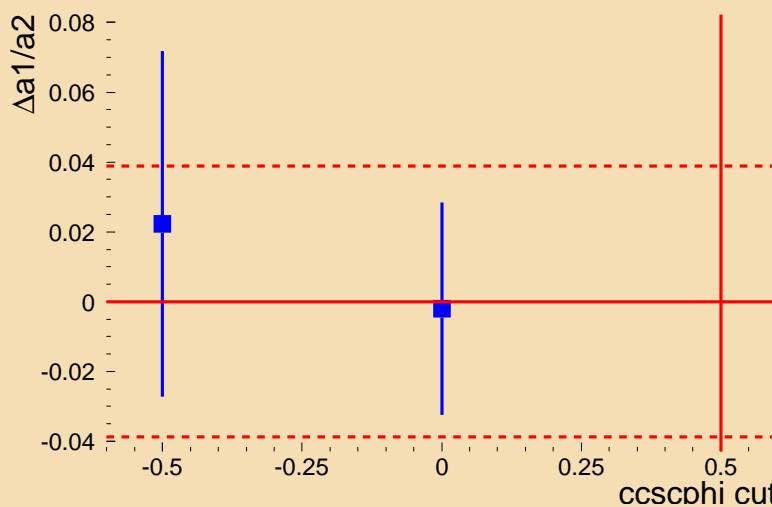
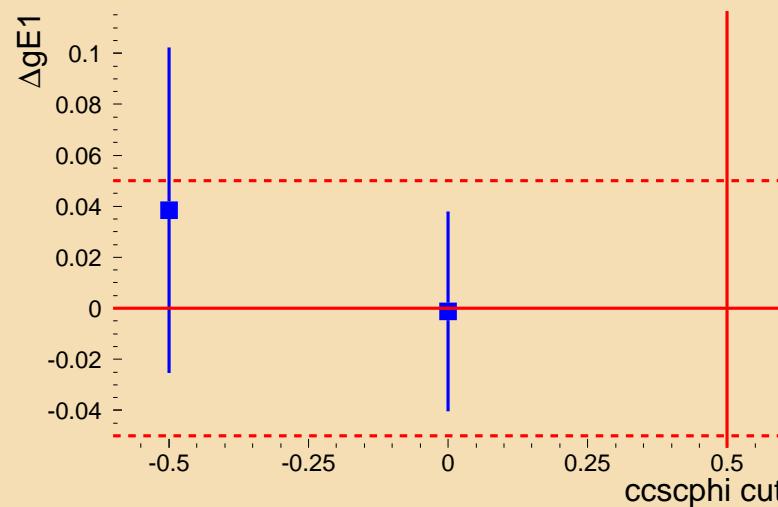
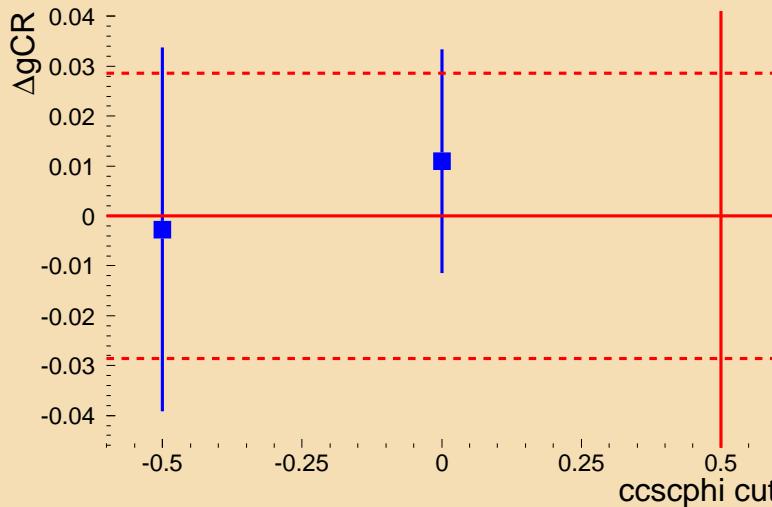
Source	Uncertainty on the Parameter			
	Δg_{CR}	$\Delta \frac{ g_{E_1} }{ g_{M_1} }$	$\Delta \frac{a_1}{a_2}$	Δg_{M_1}
Rad. Corr.	0.0	0.0	0.0	0.0
Background	0.01	0.008	0.022	0.05
Variation of Cuts	0.021	0.018	0.022	0.041
Variation of α_0	0.001	0.001	0.005	0.02
Limited MC	0.001	0.001	0.002	0.01
$\Delta\eta_{+-}$	0.002	0.0002	0.0001	0.01
$\Delta\Phi_{+-}$	0.0002	0.0005	0.0003	0.002
$\Delta\delta'$'s	0.001	0.0003	0.001	0.004
Assigned Error	0.023	0.020	0.032	0.07
Compare to Stat.	0.017	0.028	0.022	0.12

26. Various Cross-Checks

- ✓ Plus/Minus: Take events come from + or - of the $\sin\phi \cdot \cos\phi$ distribution and fit separately.
- ✓ East/West: Events come from east/west beam.
- ✓ Cut by Z_{vtx} in half: *ditto*
- ✓ Cut by $E_{\pi^+\pi^-e^+e^-}$ in half: *ditto*

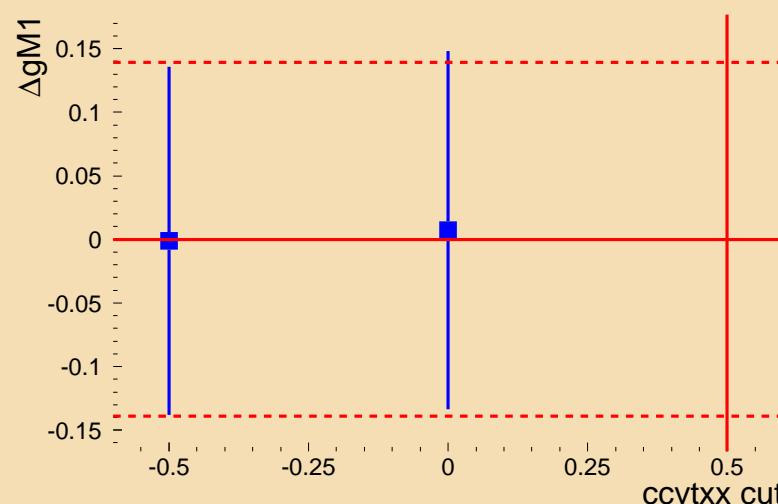
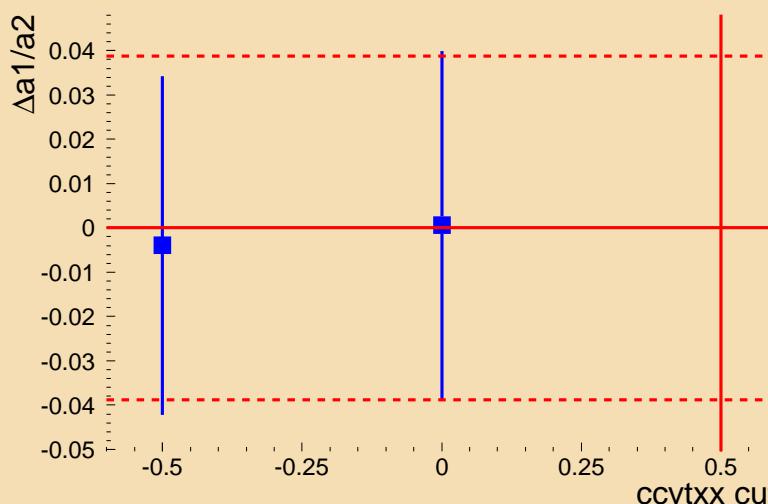
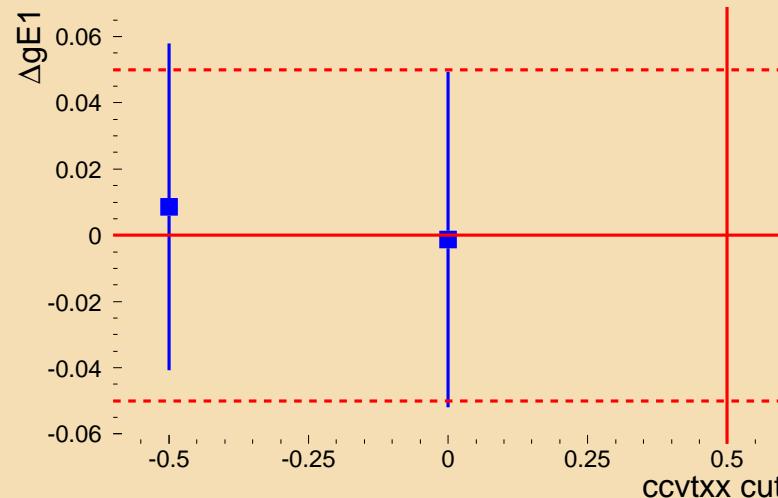
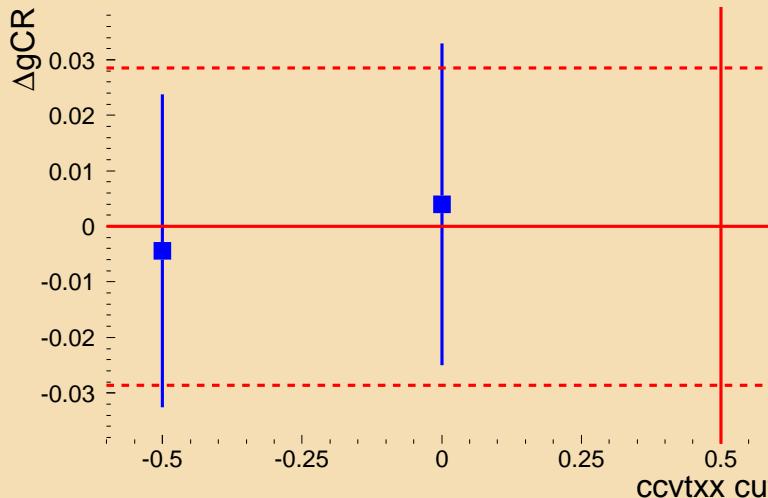
27. Divide Data Set in Two Halfes by $\sin\phi \cdot \cos\phi$

Estimated shifts: $\Delta g_{CR}=0.01$, $\Delta g_{E1}=0.01$, $\Delta a_{12}=0.01$, $\Delta g_{M1}=0.01$



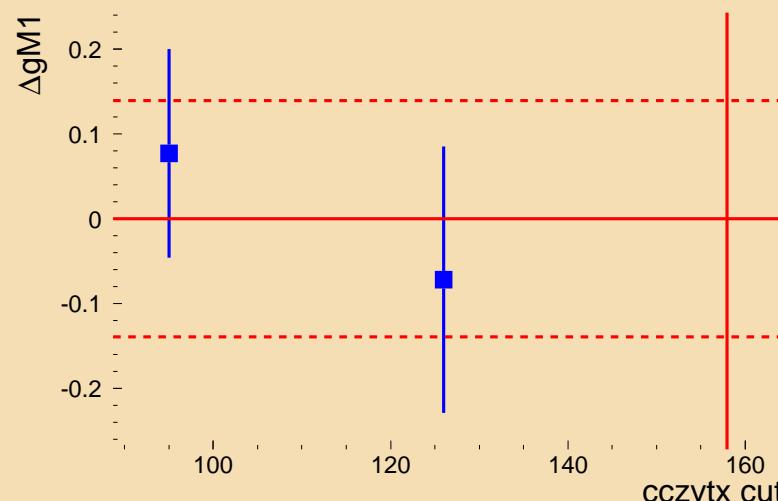
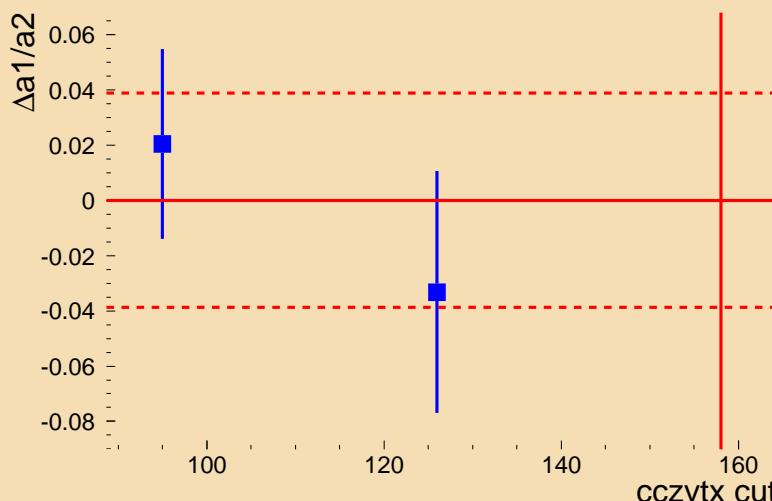
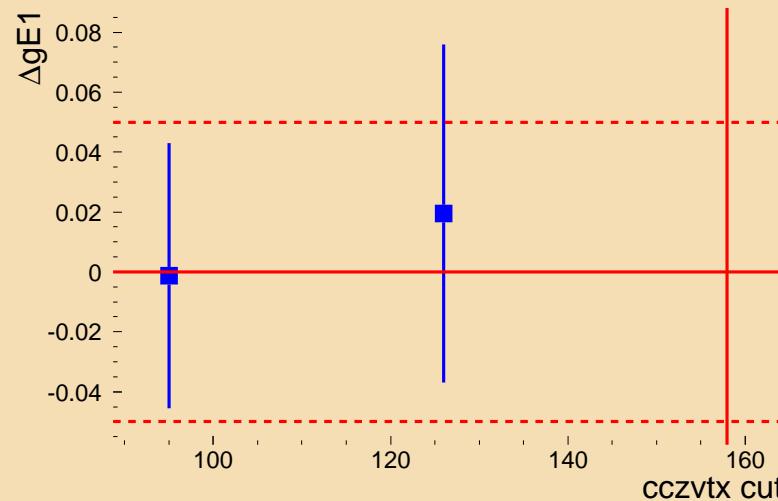
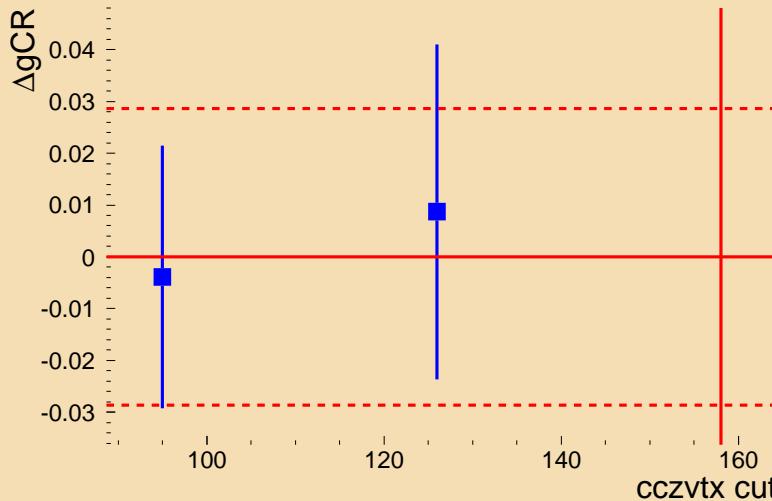
28. Divide Data Set in Two Halves by Beam

Estimated shifts: $\Delta g_{CR}=0.01$, $\Delta g_{E1}=0.01$, $\Delta a_{12}=0.01$, $\Delta g_{M1}=0.01$



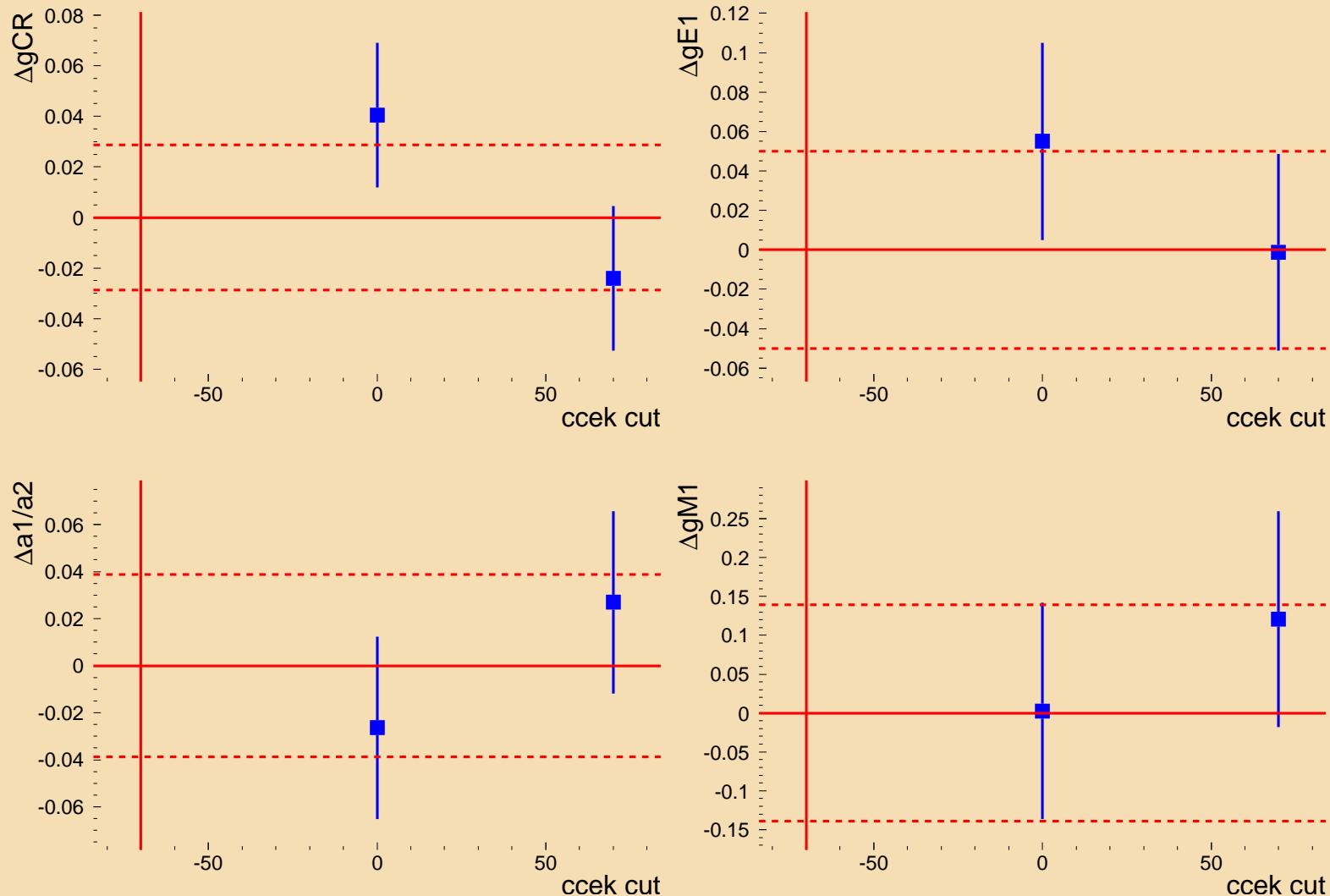
29. Divide Data Set in Two Halves by Z_{vtx}

Estimated shifts: $\Delta gCR=0.01$, $\Delta gE1=0.01$, $\Delta a12=0.01$, $\Delta gM1=0.01$



30. Divide Data Set in Two Halfes by $E_{\pi^+\pi^-e^+e^-}$

Estimated shifts: $\Delta g_{CR}=0.01$, $\Delta g_{E1}=0.01$, $\Delta a_{12}=0.01$, $\Delta g_{M1}=0.01$



31. Updated Fit Results

- ✓ Results of the 4 parameter fit for the total KTeV dataset

$$g_{\text{CR}} = 0.163 \pm 0.017(\text{stat}) \pm 0.023(\text{syst})$$

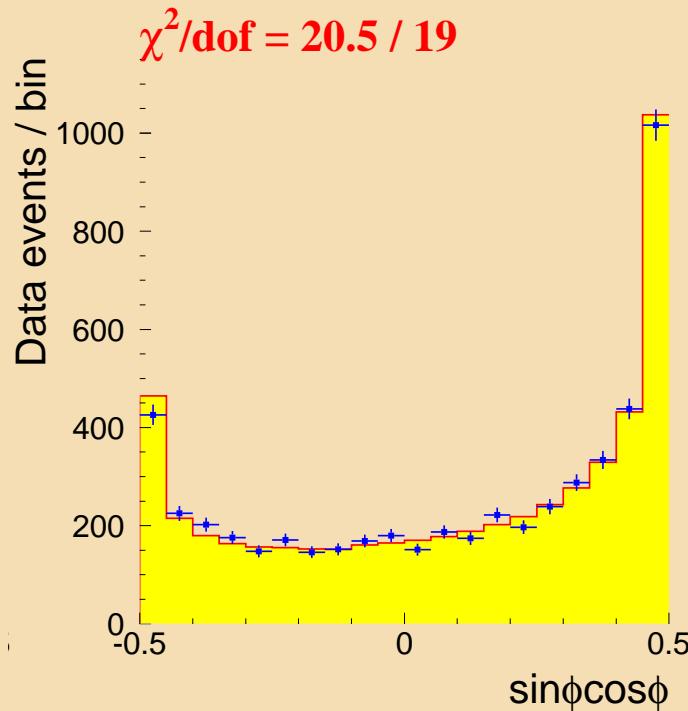
$$\frac{|g_{E_1}|}{|g_{M_1}|} < 0.04(90\% \text{CL})$$

$$\frac{a_1}{a_2} = -0.744 \pm 0.022(\text{stat}) \pm 0.032(\text{syst})$$

$$\tilde{g}_{M_1} = 1.11 \pm 0.12(\text{stat}) \pm 0.07(\text{syst})$$

- ✓ These are the **final numbers**.
- ✓ Use these values for the asymmetry calculation.

32. New Value For The Asymmetry

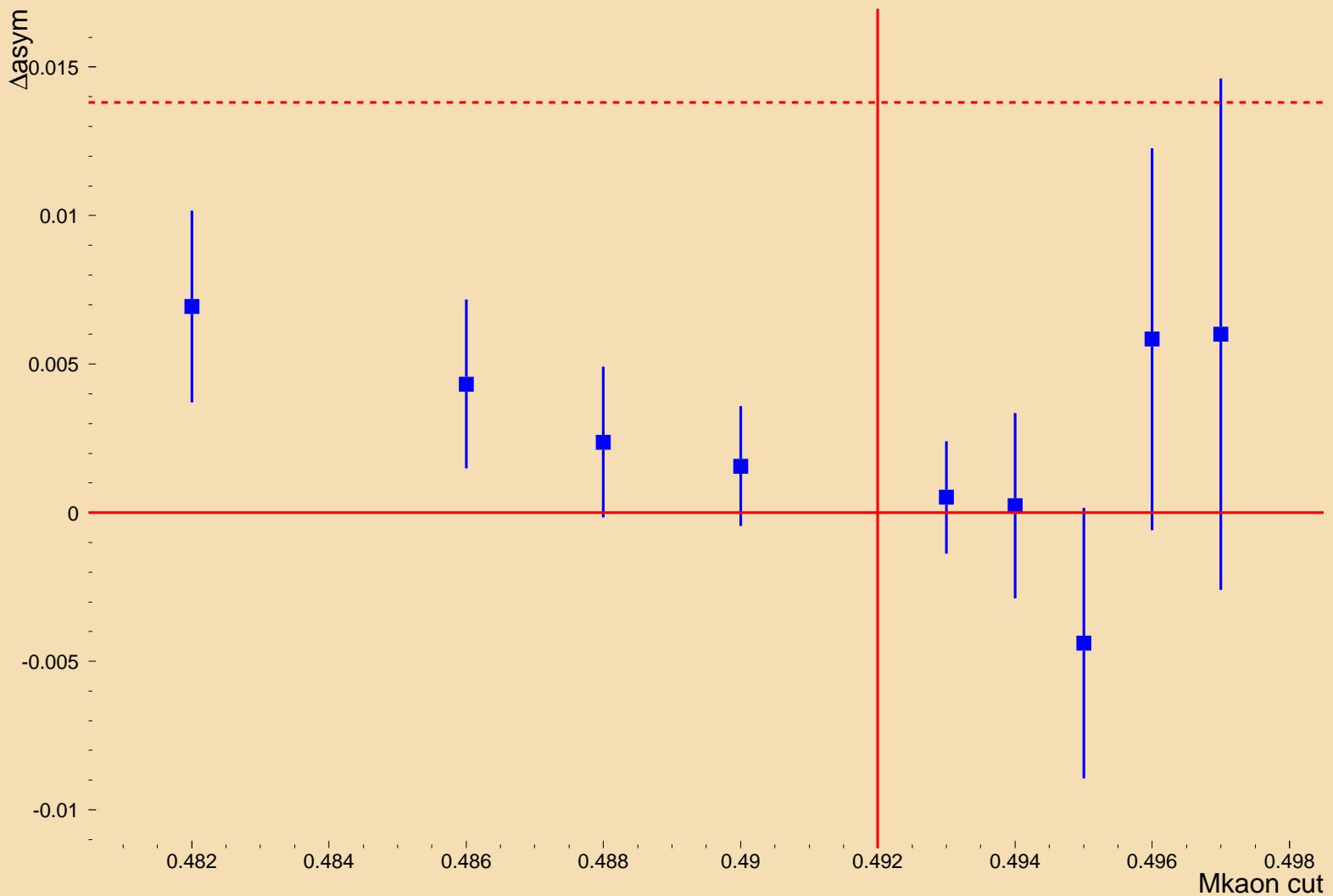


✓ $\mathcal{A} = (13.71 \pm 1.38(\text{stat}))\%$

✓ Now work out systematic uncertainty: matrix element **parametrization**, **background** and **cut variation**.

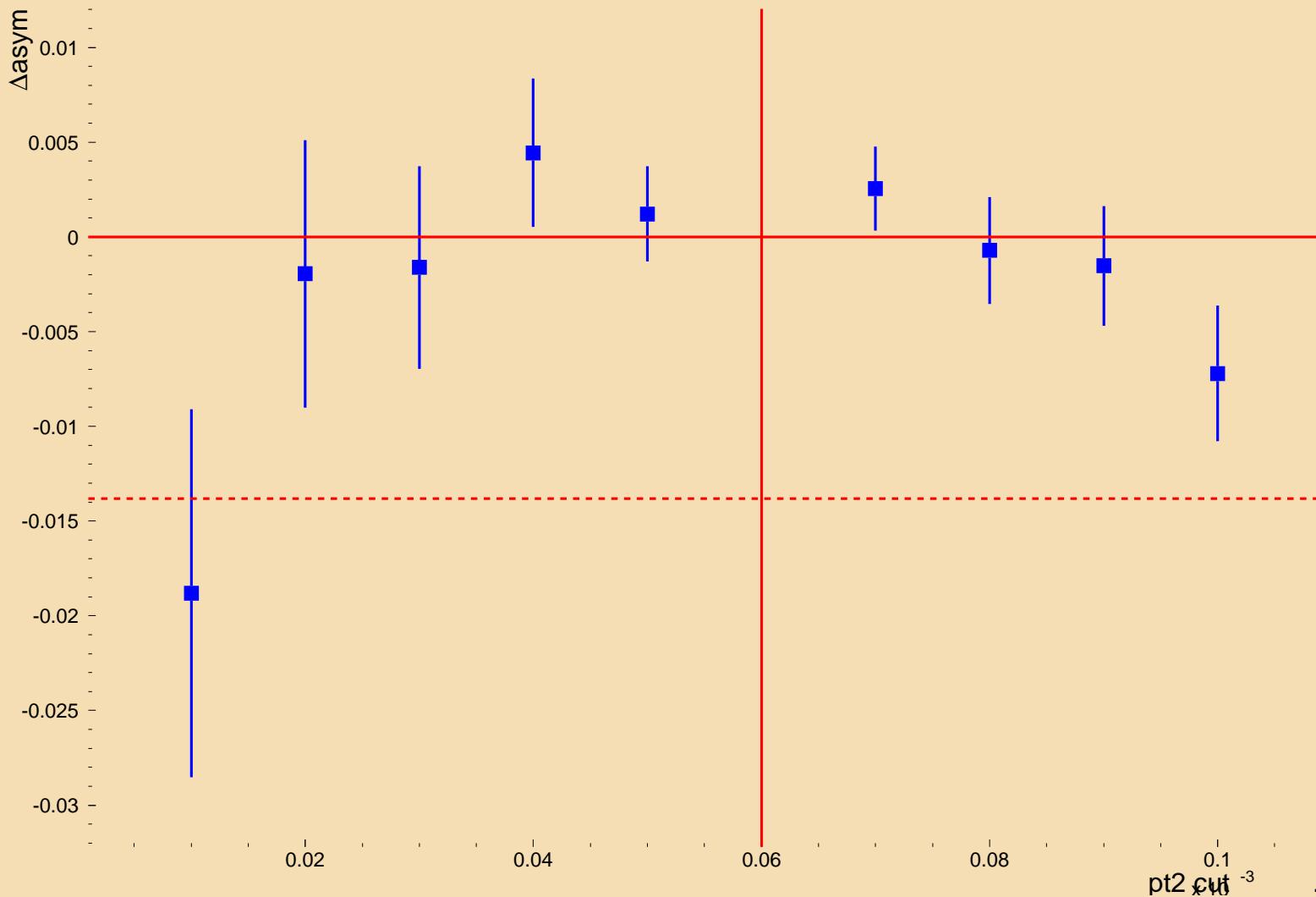
33. Vary Kaon Mass Cut (and calculate \mathcal{A})

Estimated shift: $\Delta_{\text{asym}}=0$



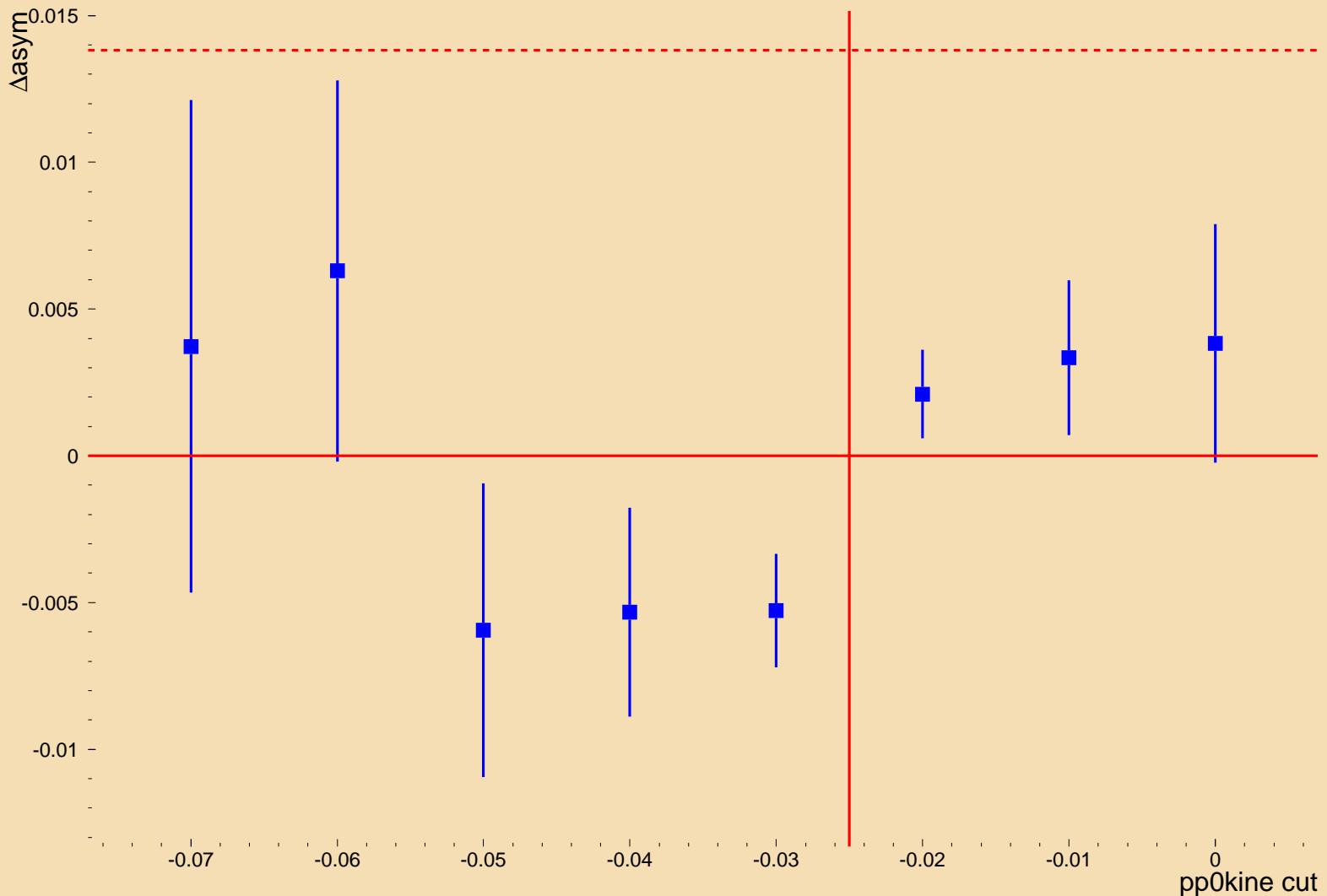
34. Vary P_T^2 Cut (and calculate \mathcal{A})

Estimated shift: $\Delta_{\text{asym}}=0$



35. Vary p0kine Cut (and calculate \mathcal{A})

Estimated shift: $\Delta\text{asym}=0.003$

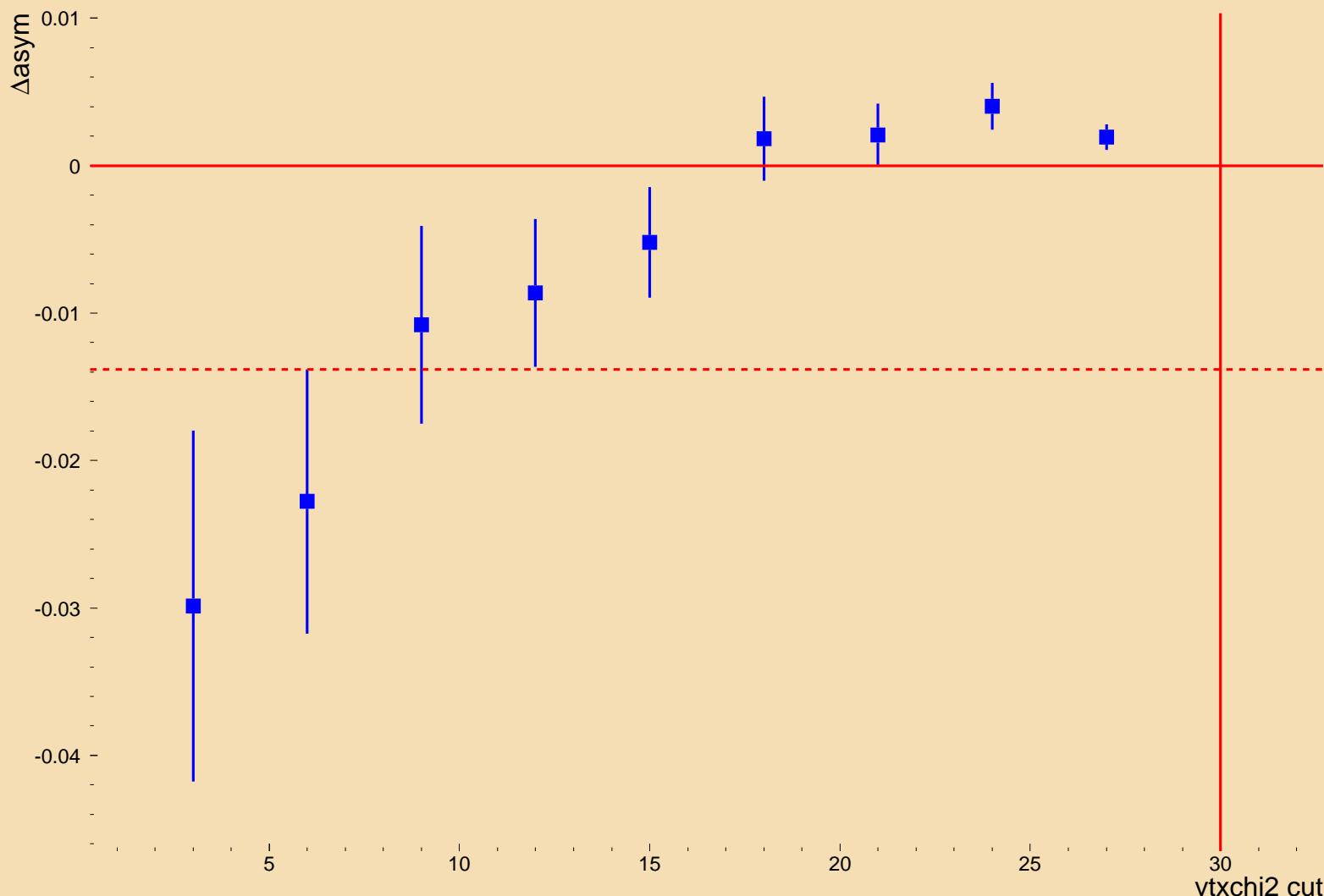


36. Summarize Background Systematics (\mathcal{A})

Cut	Estimated $\Delta\mathcal{A}$, %
Kaon Mass	0.0
P_T^2	0.0
pp0kine	0.3
Assigned Error	0.3
Compare to Stat.	1.38

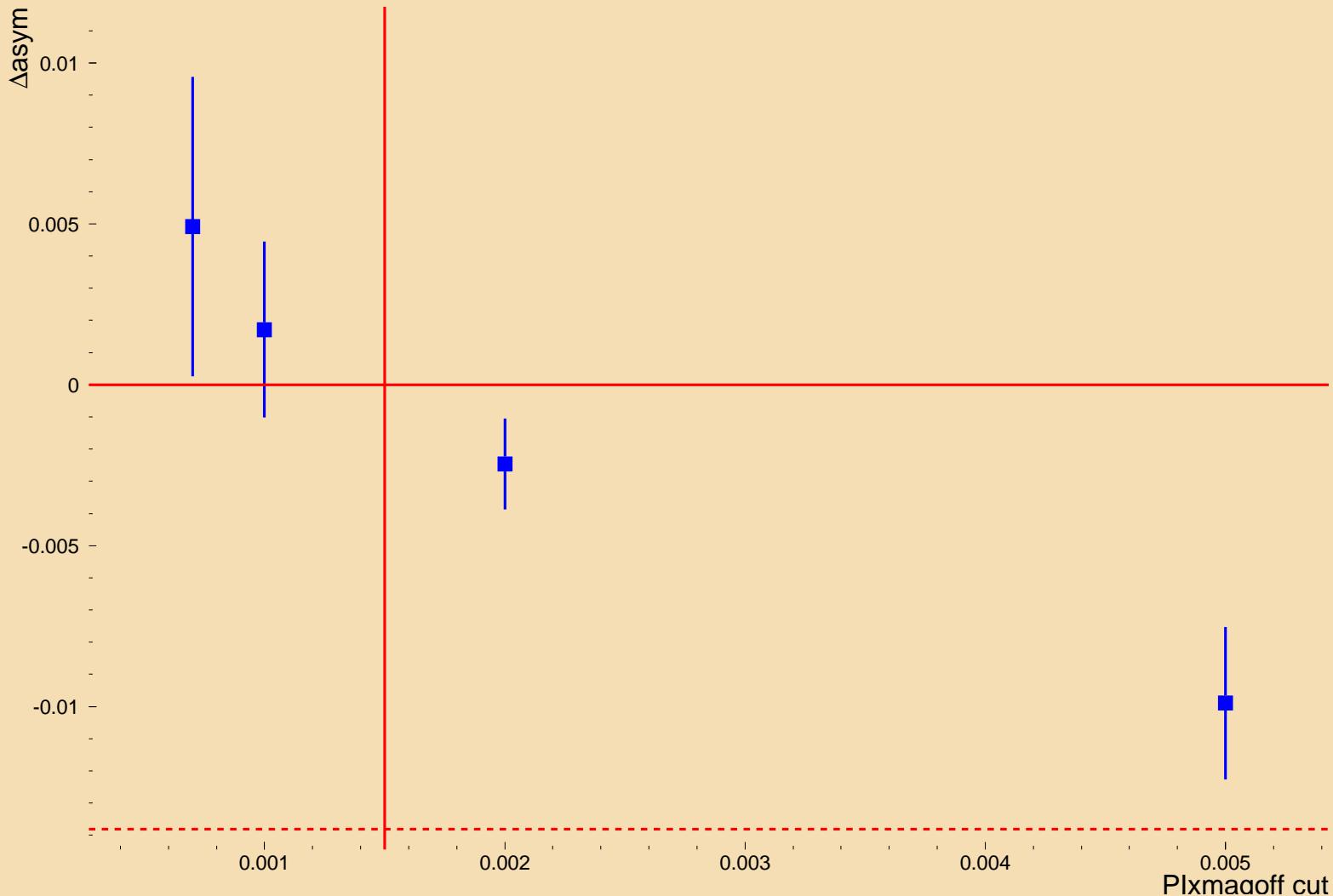
37. Vary χ^2_{vtx} Cut (and calculate \mathcal{A})

Estimated shift: $\Delta\text{asym}=0.004$



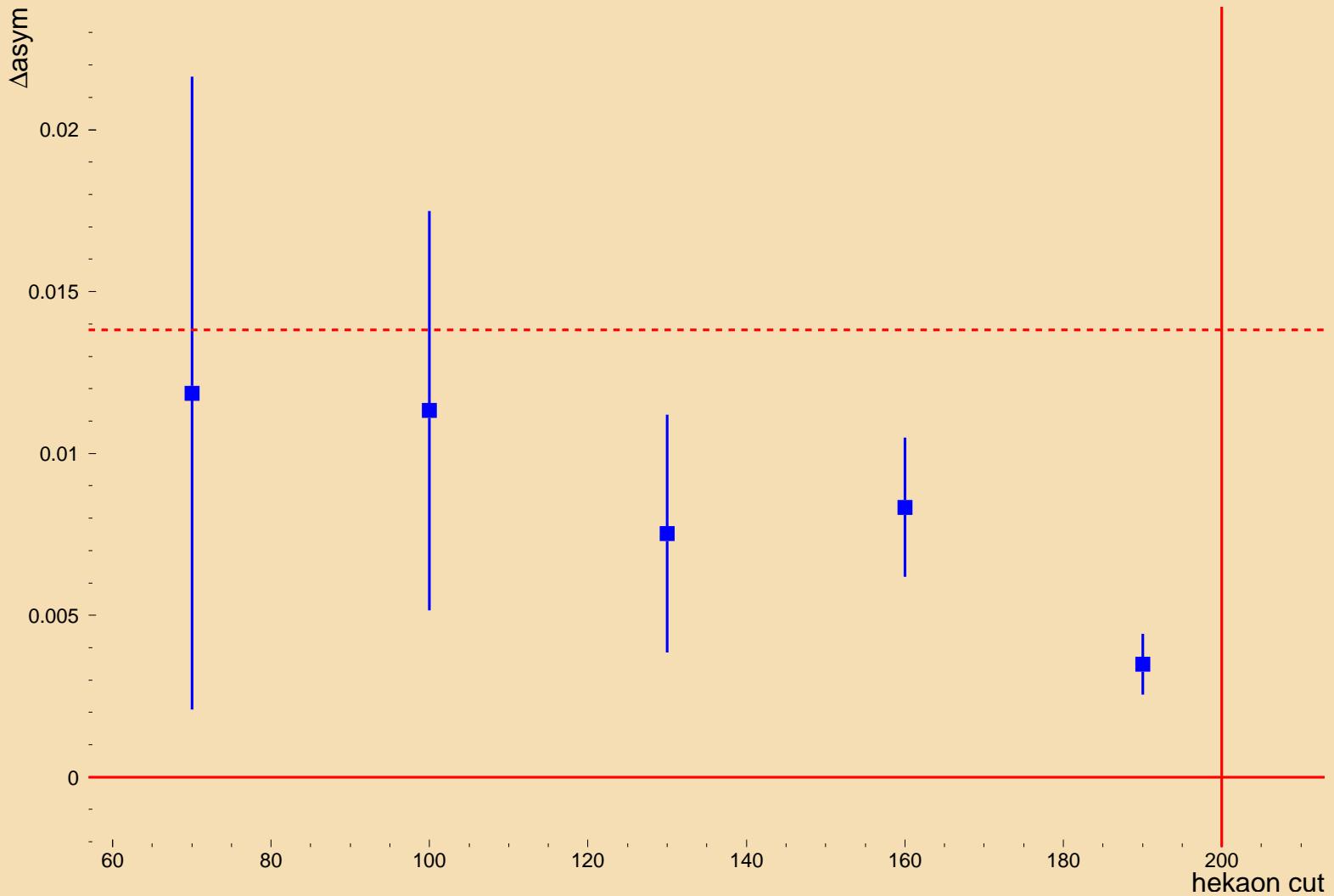
38. Vary Cut on the Magnet Offset of Pion (\mathcal{A})

Estimated shift: $\Delta\text{asym}=0.003$



39. Vary Kaon Energy Cut (and calculate \mathcal{A})

Estimated shift: $\Delta\text{asym}=0.005$



40. Summarize Cut Variation Systematics For \mathcal{A}

Cut	Estimated $\Delta\mathcal{A}$, %
χ^2_{vtx}	0.4
magoff for π' s	0.3
E_{kaon}	0.5
Assigned Error	0.71
Compare to Stat.	1.38

41. Summarize All Systematics for \mathcal{A}

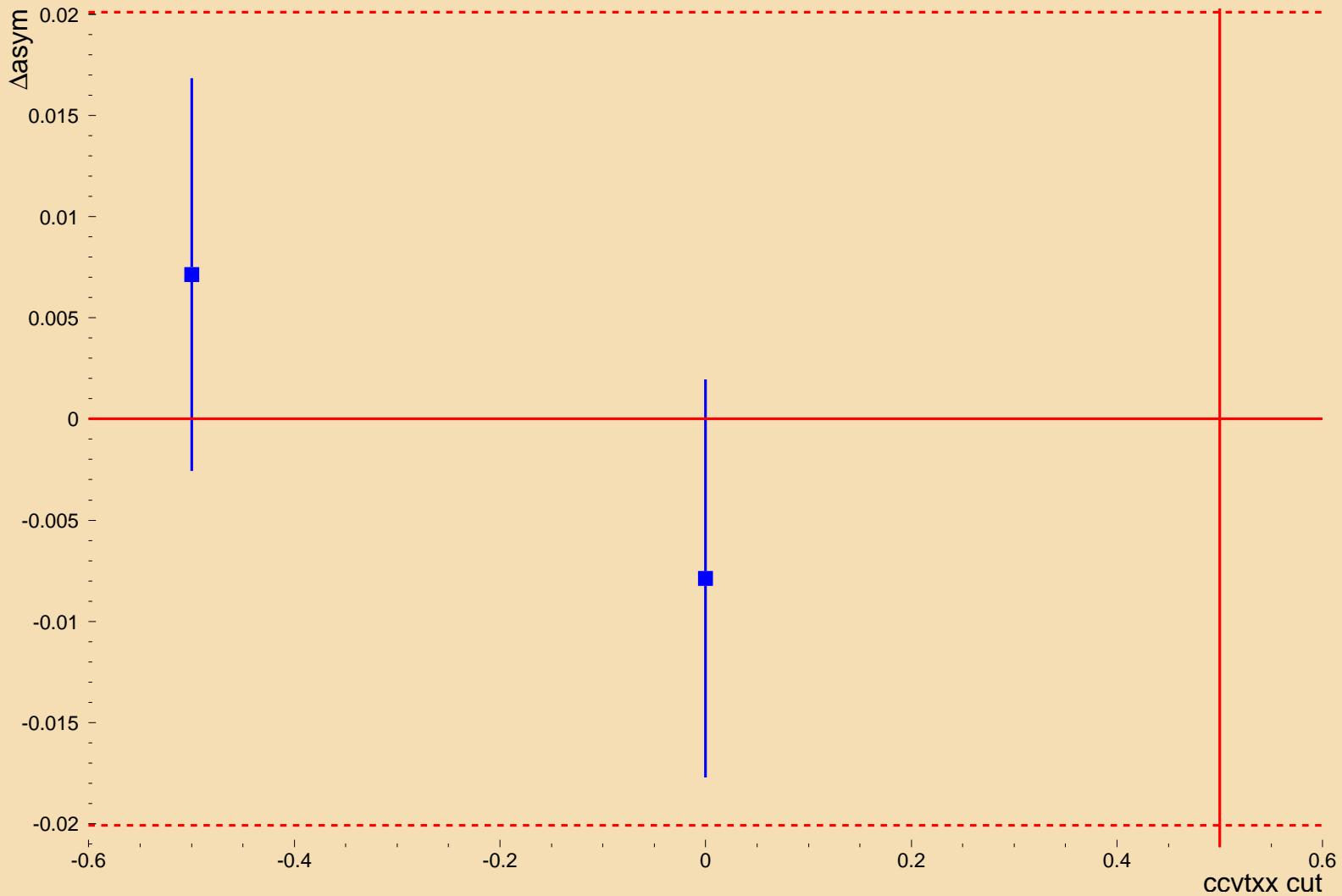
Source	Estimated $\Delta\mathcal{A}$, %
Background	0.3
Variation of Cuts	0.71
$\Delta\eta_{+-}$	0.163
$\Delta\Phi_{+-}$	0.111
$\Delta\delta'$ s	0.325
Δg_{E1}	0.326
Δg_{M1}	0.335
Δg_{CR}	0.335
Assigned Error	1.46
Compare to Stat.	1.38

42. Various Cross-Checks (\mathcal{A})

- ✓ *the same cross-checks as for the fits!*
- ✓ *calculate the asymmetry for each cross check*

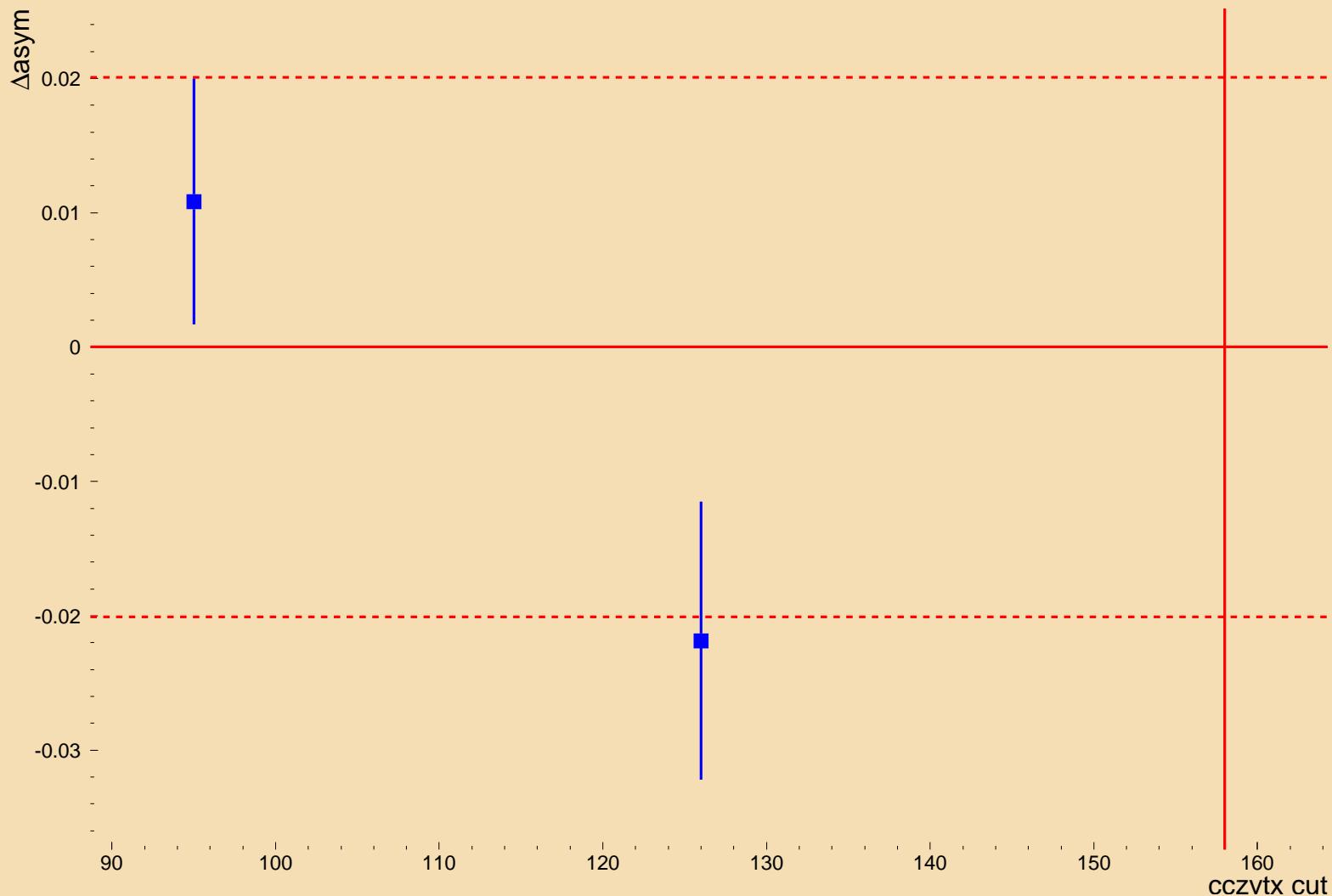
43. Divide Data Set in Two Halves by Beam

Estimated shift: $\Delta_{\text{asym}}=0$



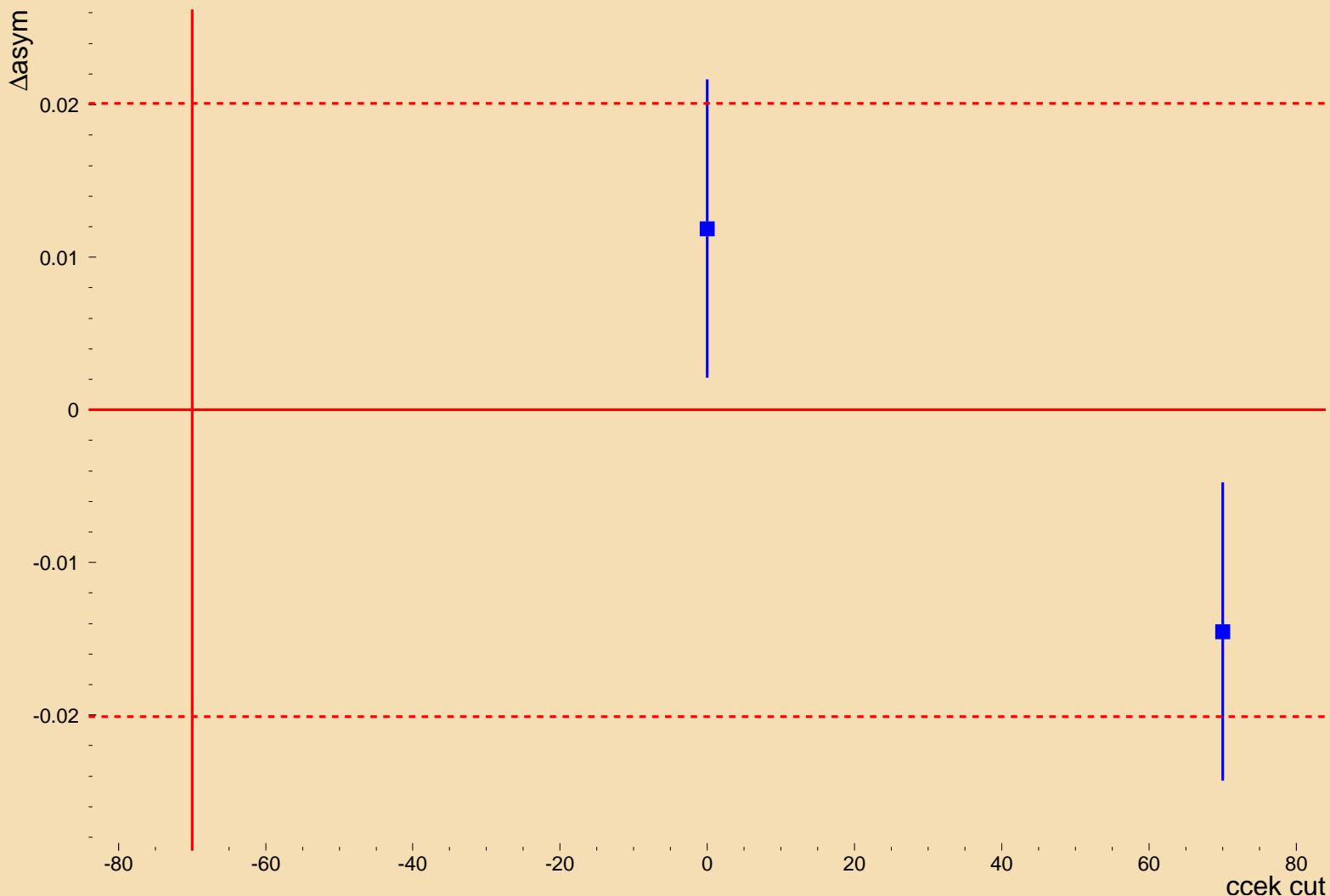
44. Divide Data Set in Two Halfes by Z_{vtx}

Estimated shift: $\Delta\text{asym}=0.01$



45. Divide Data Set in Two Halfes by $E_{\pi^+\pi^-e^+e^-}$

Estimated shift: $\Delta\text{asym}=0.004$



46. Final Results

✓ Results of the **4 parameter fit** for the total KTeV dataset

$$g_{\text{CR}} = 0.163 \pm 0.017(\text{stat}) \pm 0.023(\text{syst})$$

$$\frac{|g_{E_1}|}{|g_{M_1}|} < 0.04(90\%\text{CL})$$

$$\frac{a_1}{a_2} = -0.744 \pm 0.022(\text{stat}) \pm 0.032(\text{syst})$$

$$\tilde{g}_{M_1} = 1.11 \pm 0.12(\text{stat}) \pm 0.07(\text{syst})$$

✓ Results **calculated** from the FF's

$$\langle |g_{M_1}| \rangle = 0.739 \pm 0.134$$

$$\langle R_K^2 \rangle = (-0.077 \pm 0.008(\text{stat}) \pm 0.011(\text{syst})) fm^2$$

✓ **Asymmetry** and Branching Ratio

$$\mathcal{A} = (13.71 \pm 1.38(\text{stat}) \pm 1.46(\text{syst}))\%$$

$$\mathcal{B} = (3.67 \pm .07(\text{stat}) \pm ...(\text{syst})) \cdot 10^{-7}$$